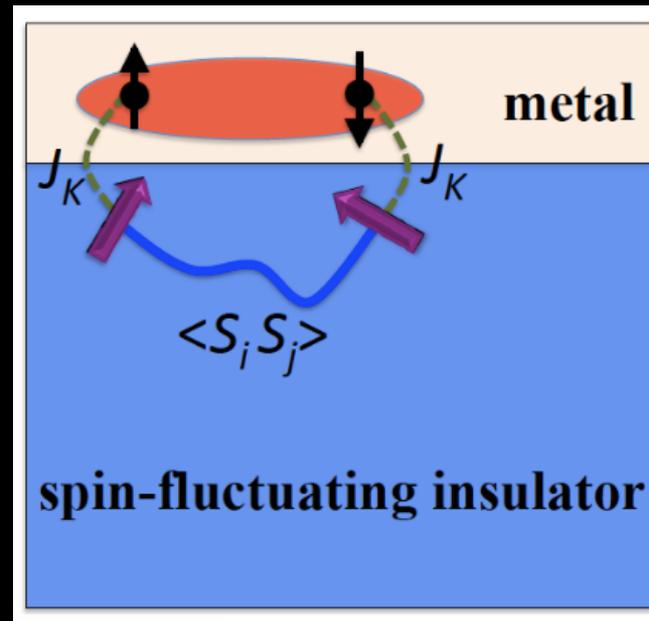


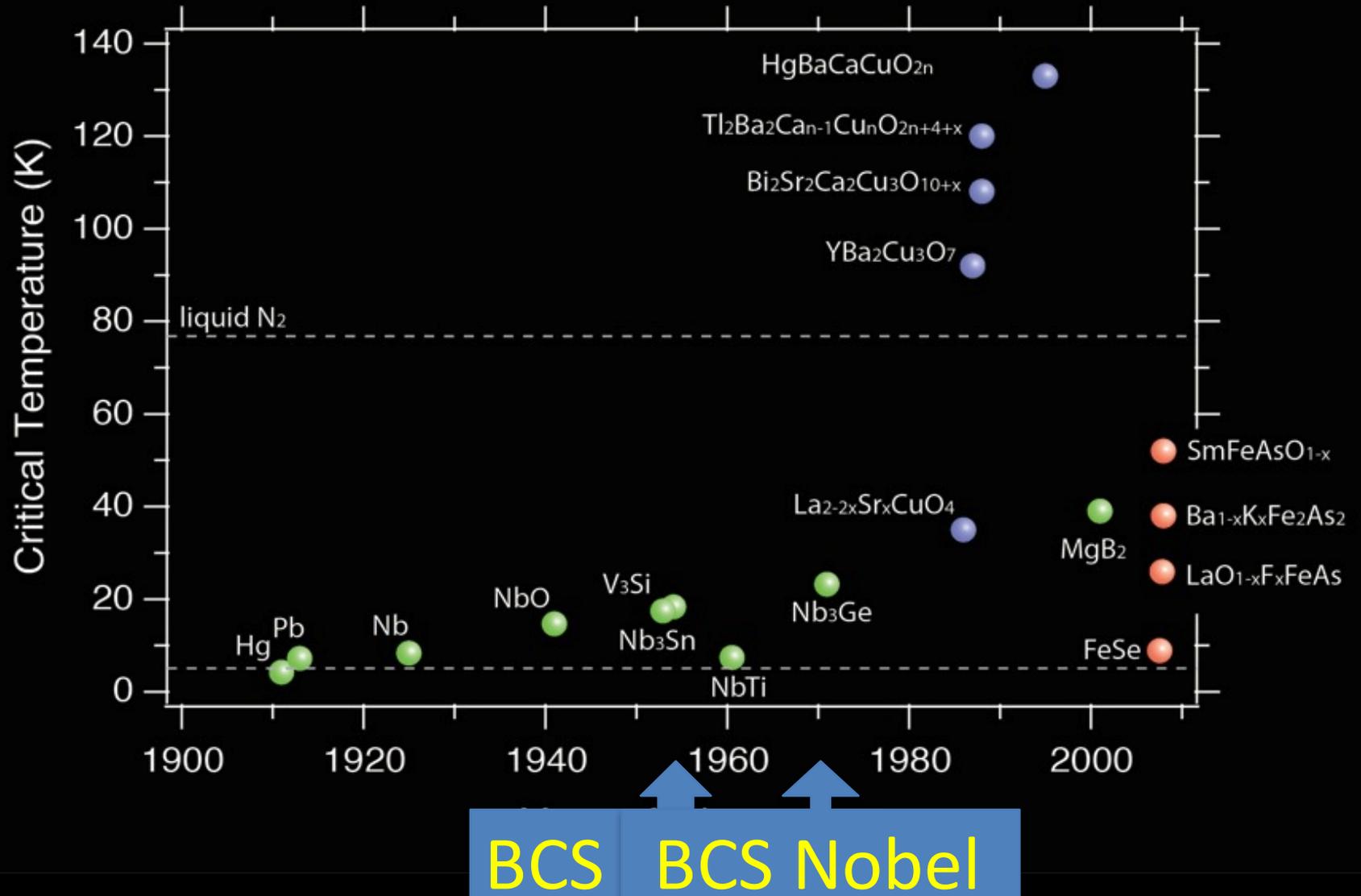
# Topological Superconductivity in Metal/Quantum-Spin-Ice Heterostructures



Eun-Ah Kim (Cornell)

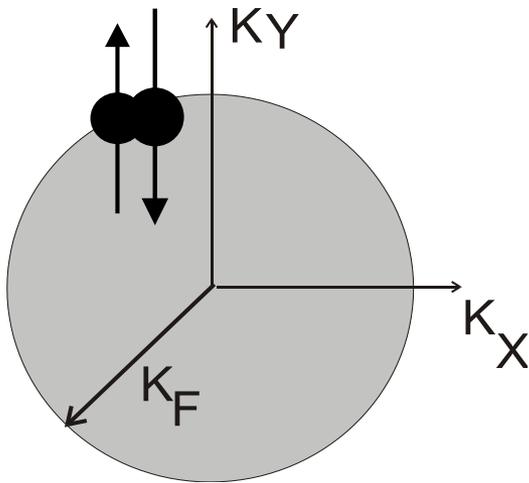
Q1. Can a theory guide discovery  
of a new superconductor?

# History of Serendipitous discoveries



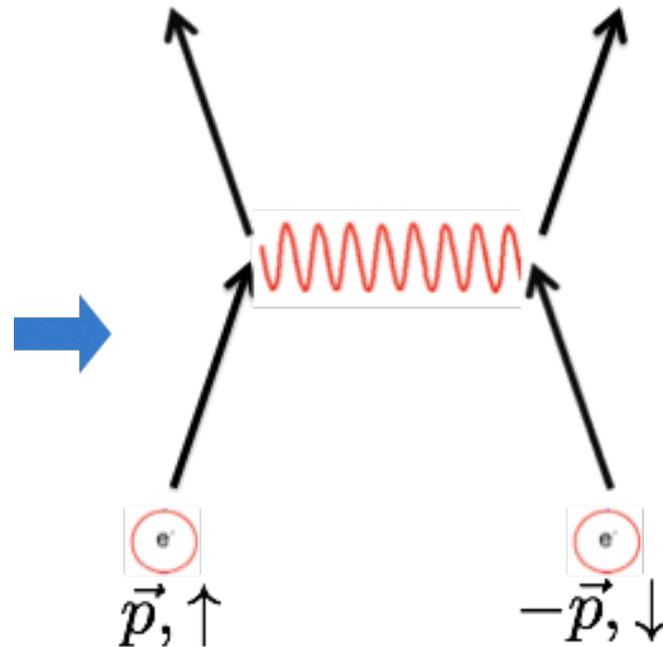
# BCS theory

Normal State (Metal)



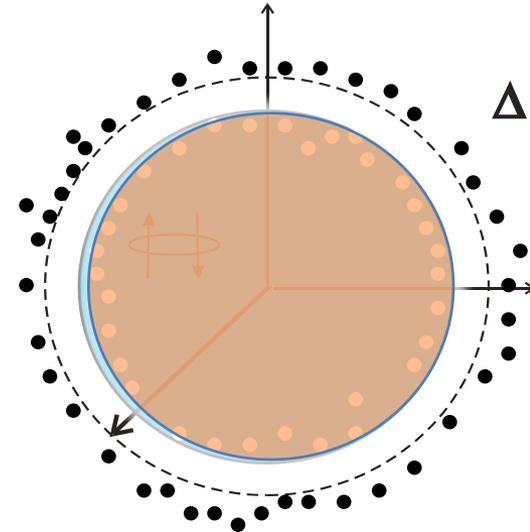
Degenerate  
 $\sim$ free electron gas

Weak coupling instability



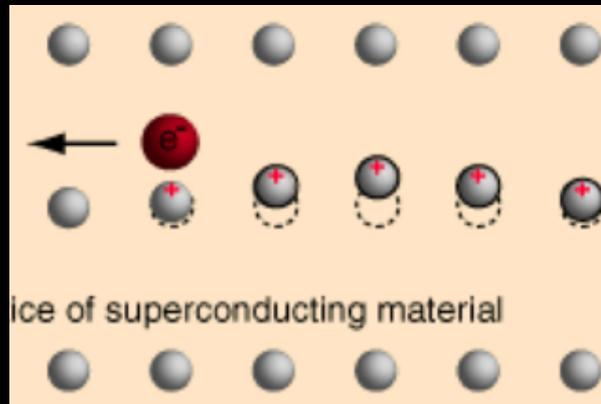
Electrons interacting with  
Lattice Vibration Mode

Superconductor



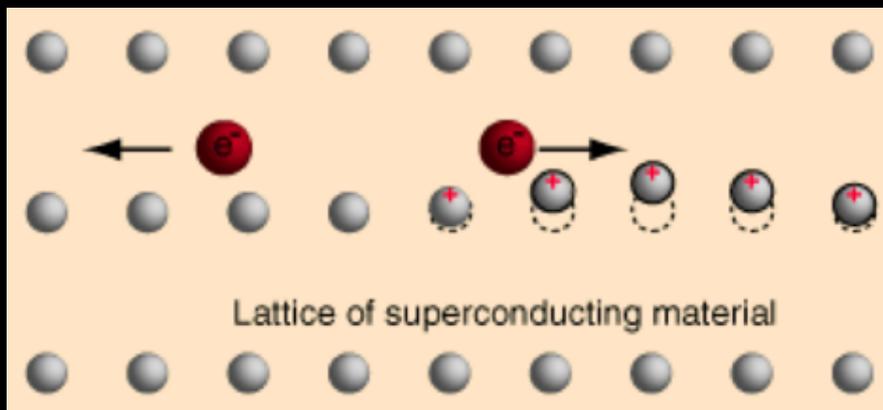
Gas of Cooper pairs

# Magic ingredient of BCS theory



separation of  
scales:

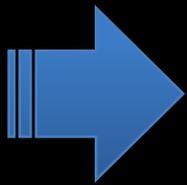
$$\frac{\omega_D}{E_F} \ll 1$$



# Migdal-Eliashberg theory

- Organize diagrams using  $\frac{\omega_D}{E_F} \ll 1$
- Sum infinite number of leading diagrams.
- Result boils down to BCS mean-field theory

when  $\lambda \equiv V E_F < 1$



BCS mean-field theory is exact!!

# Electronic (non-phonon) Mechanisms

- Necessary for exotic (non-s-wave) SC.

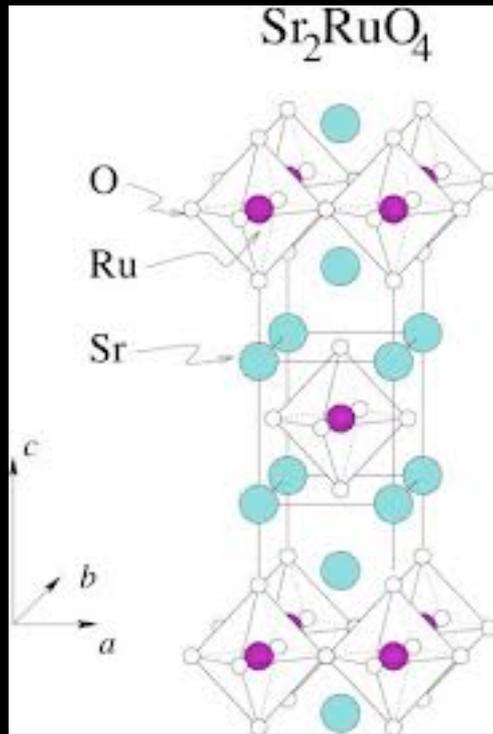
- Give up separation of scales:  $\frac{E_F}{E_F} = 1$



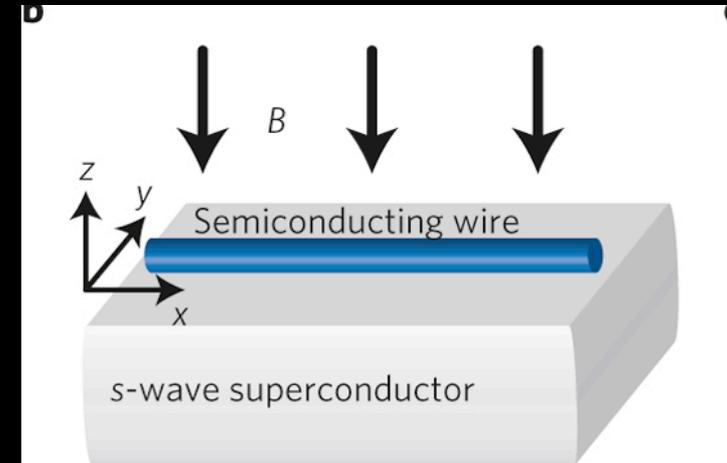
Untractable problem out of  
reach of BCS mean-field theory

# Q. Topological Superconductor material?

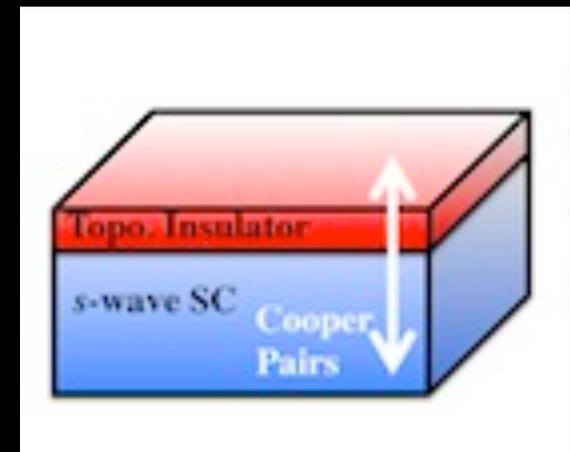
Bulk



1D proximity



2D proximity?



Anderson's proposal

: dope a quantum spin liquid (QSL)

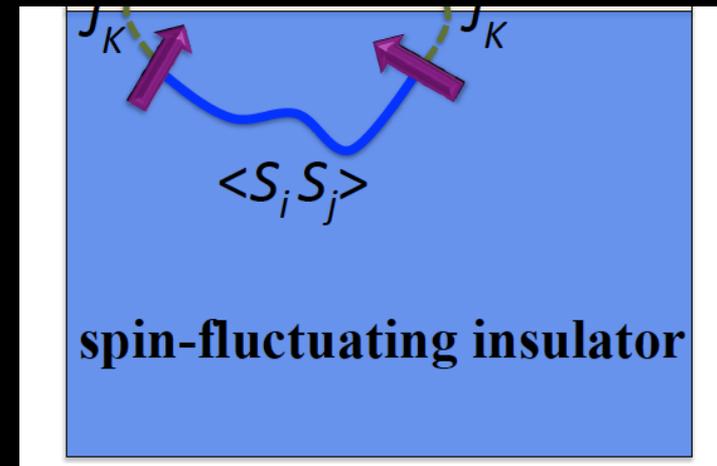
# QSL

- Definition:

- No magnetic order at  $T=0$   $\langle \vec{s}_{\vec{q}} \rangle = 0$

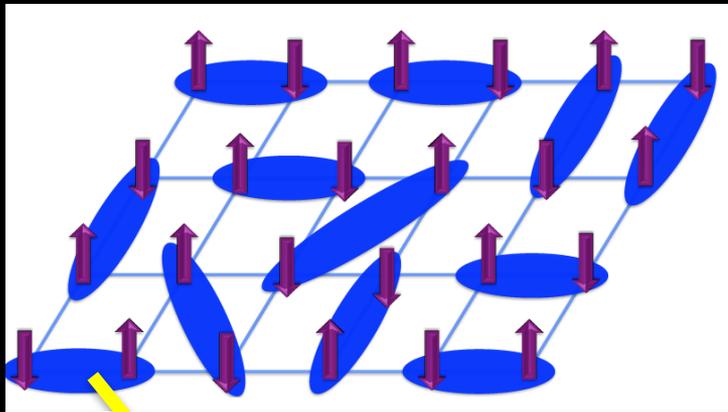
- Dynamic fluctuation  $\langle \vec{s}_{-\vec{q}}(t) \vec{s}_{\vec{q}}(0) \rangle \neq 0$

- Spin's are entangled.



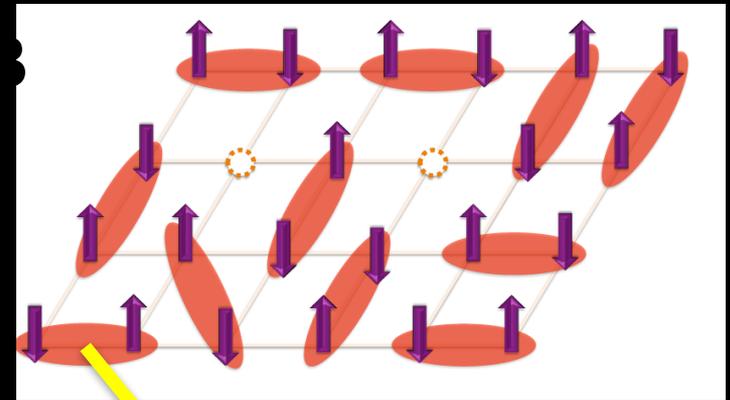
# Anderson's conjecture

QSL = Resonating  
Valence Bond state



RVB singlet

→  
Doping



Cooper pair = mobile  
RVB singlet

# Challenges against Anderson's conjecture

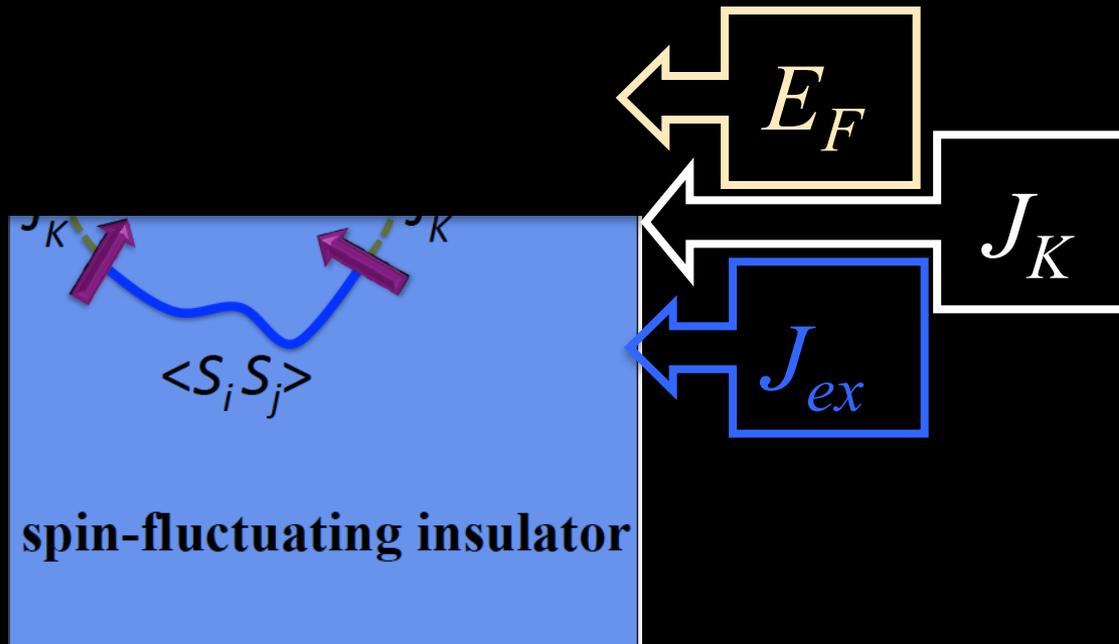
- Experimental:
  - Hard to dope QSL
- Theoretical:
  - No controlled theory
  - Predictions are based on faith and hope...

Q2. Can we exploit the spin entanglement in QSL for SC?

# A new approach

Keep the QSL and borrow the spin entanglement:

## Heterostructure!



# Challenges against Anderson's conjecture

- Experimental:
  - Hard to dope QSL
- Theoretical:
  - No controlled theory
  - Predictions are based on faith and hope...

# Advantages of the Heterostructure route

- Experimental:
  - Accessible to current MBE technology
- Theoretical:
  - Separation of scales:  $J_{ex}/E_F < 1$
  - Predictions are based on faith and hope...

# Strategy

Effective Field theory



A microscopic theory for a  
concrete proposal

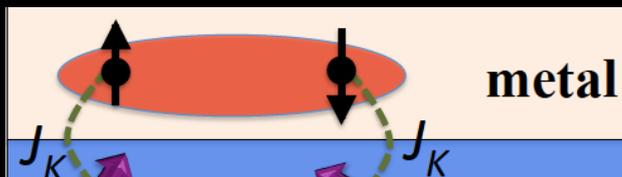


Persuade experimentalists

# Effective Field Theory

# The starting point: Kondo-Heisenberg

$$H_c = \sum_{\mathbf{k}\alpha} \left( \frac{\hbar^2 k^2}{2m} - E_F \right) \psi_\alpha^\dagger(\mathbf{k}) \psi_\alpha(\mathbf{k})$$



$E_F$

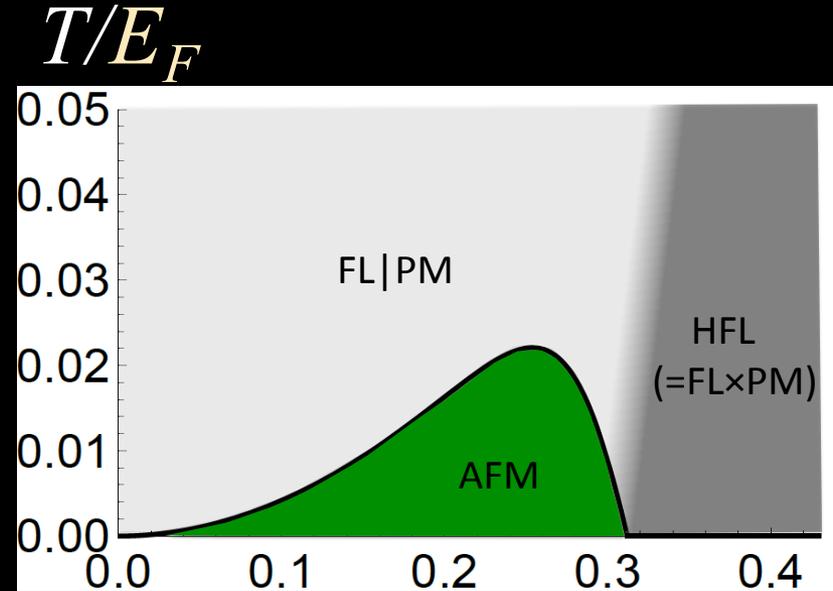
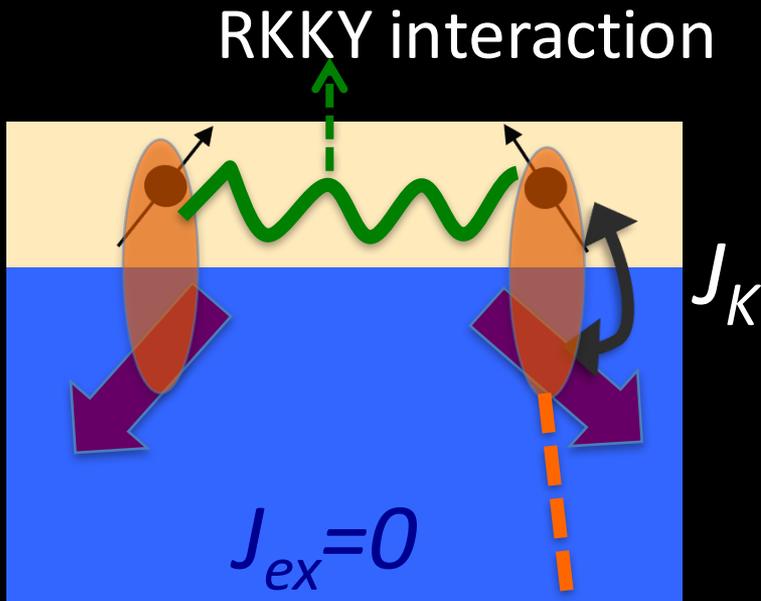
$J_K$

$$H_K(t) = J_K v_{\text{cell}} \sum_{a\alpha\beta} \int d^2\mathbf{r} \psi_\alpha^\dagger(\mathbf{r}) \sigma_{\alpha\beta}^a \psi_\beta(\mathbf{r}) S_a(\mathbf{r}_\perp = \mathbf{r}, z = 0, t)$$

spin-fluctuating insulator

$J_{ex}$

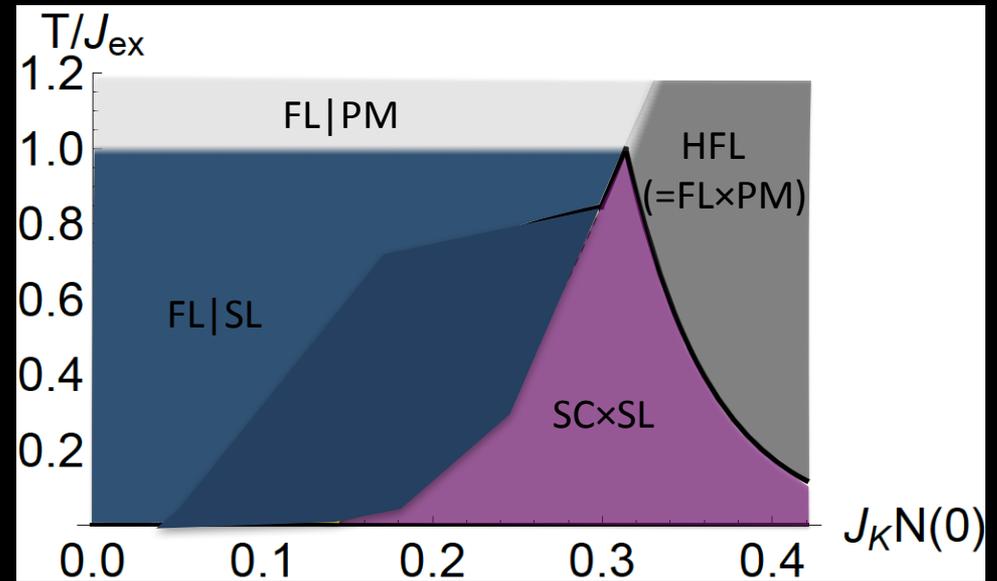
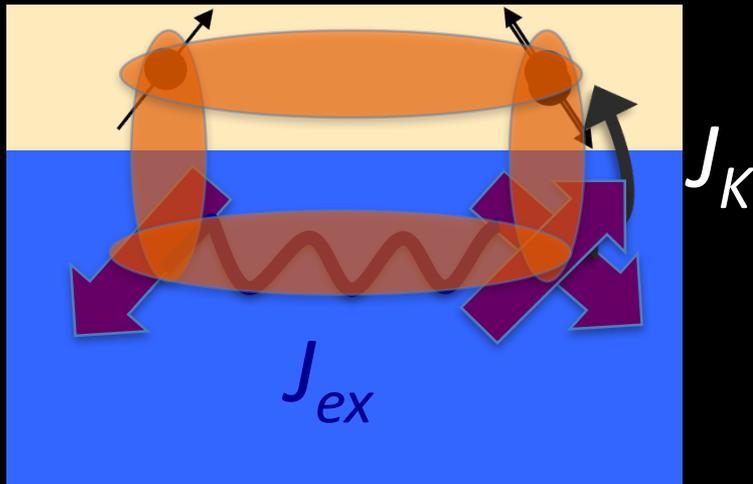
# The starting point: Kondo



The Doniach PD (1977)

Kondo-singlet: Heavy Fermi Liquid  
= Fermi liquid x Paramagnet

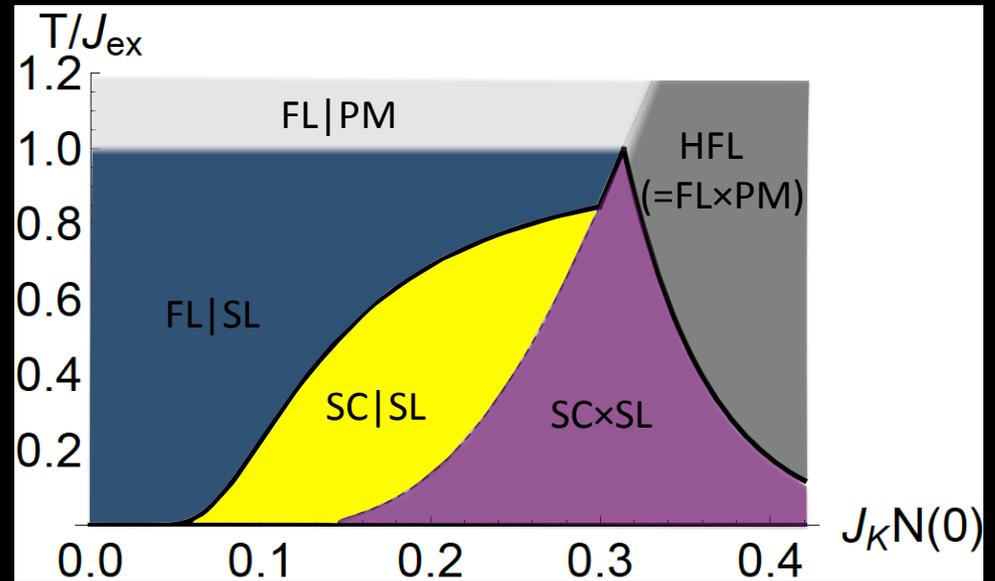
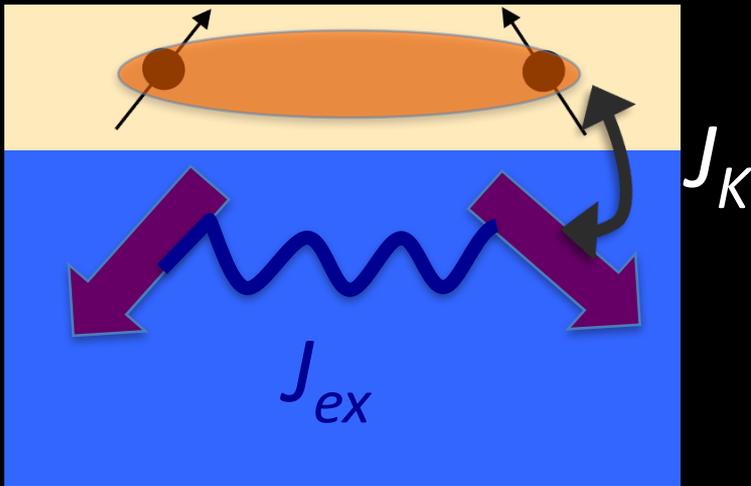
# The starting point: Kondo-Heisenberg



The PD at small  $J_{ex}$  large  $J_K$

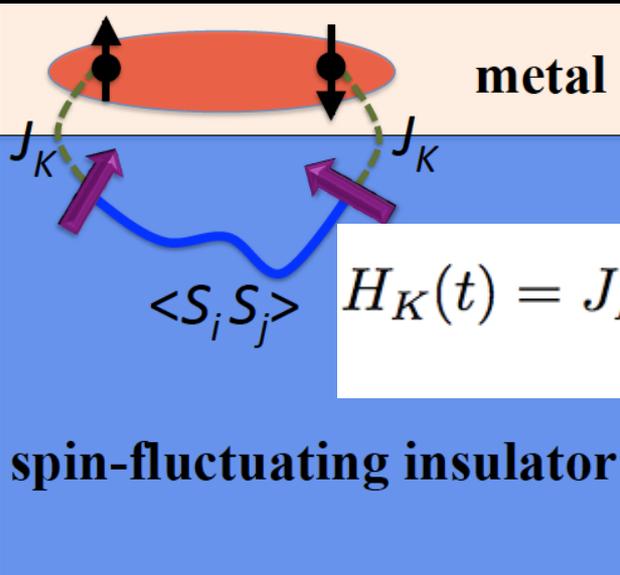
Coleman & Andrei (1989), Senthil, Sachdev, Vojta (2003)

# Heterostructure: Focus on small $J_K/E_F$



J.-H. She, C. Kim, C. Fennie, M. Lawler, EAK (2015)

# Effective Theory for $J_K/E_F \ll 1$



$$H_c = \sum_{\mathbf{k}\alpha} \left( \frac{\hbar^2 k^2}{2m} - E_F \right) \psi_\alpha^\dagger(\mathbf{k}) \psi_\alpha(\mathbf{k})$$

$$H_K(t) = J_K v_{\text{cell}} \sum_{\alpha\beta} \int d^2\mathbf{r} \psi_\alpha^\dagger(\mathbf{r}) \sigma_{\alpha\beta}^a \psi_\beta(\mathbf{r}) S_a(\mathbf{r}_\perp = \mathbf{r}, z = 0, t)$$

- Integrate out spins  $\gg$  Effective e-e interaction

$$H_{\text{int}}(t) = (J_K^2 v_{\text{cell}}^2 / \hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

$$s_a(\mathbf{r}, t) = \sum_{\alpha\beta} \psi_\alpha^\dagger(\mathbf{r}, t) \sigma_{\alpha\beta}^a \psi_\beta(\mathbf{r}, t)$$

# spin entanglement imprints onto the effective e-e interaction

$$H_{\text{int}}(t) = (J_K^2 v_{\text{cell}}^2 / \hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

$$s_a(\mathbf{r}, t) = \sum_{\alpha\beta} \psi_{\alpha}^{\dagger}(\mathbf{r}, t) \sigma_{\alpha\beta}^a \psi_{\beta}(\mathbf{r}, t)$$

- "Designed" by the choice of QSL and its

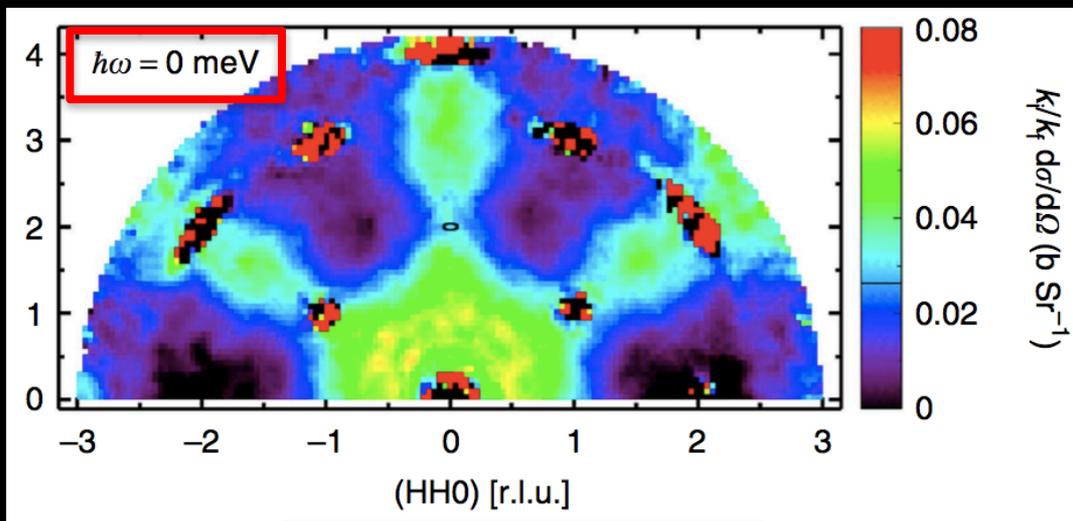
$$\mathcal{S}_{ab}(\mathbf{q}, \omega) \equiv \int dt \langle S_a(-\mathbf{q}, t) S_b(\mathbf{q}, 0) \rangle e^{-i\omega t}$$

- Experimental knowledge of  $\mathcal{S}_{ab}(\mathbf{q}, \omega)$  is sufficient

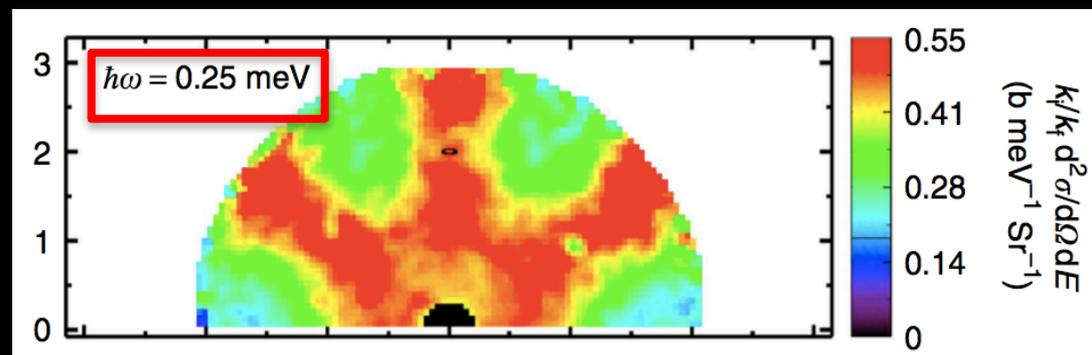
Our first pass choice of QSL:  
quantum spin ice

# Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>



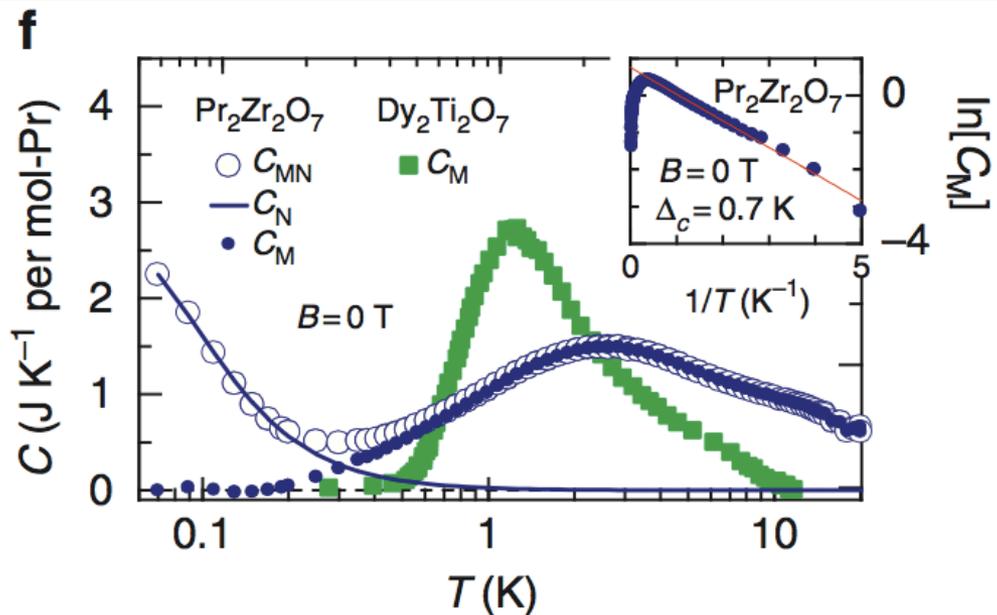
- Elastic neutron: pinch points (spin-ice like)



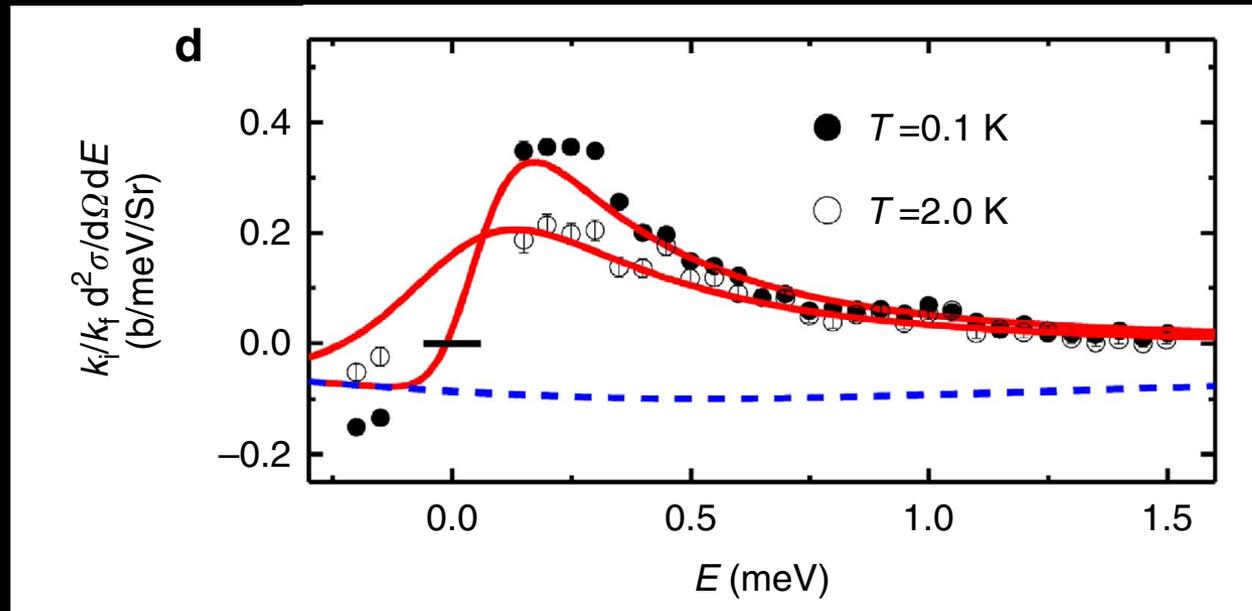
- Inelastic neutron: over 90% weight

# Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>

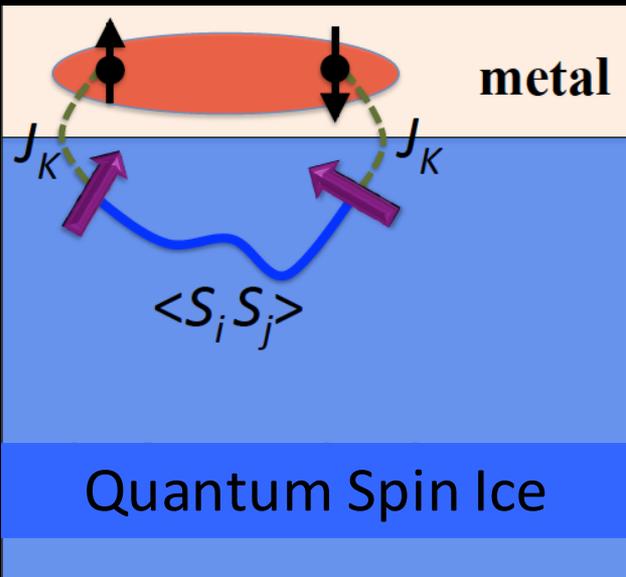


- No order down to 20mK
- Dynamic fluct. upto  $\sim 3$ K
- Gapped QSL!

Quantum fluctuations in spin-ice-like  $\text{Pr}_2\text{Zr}_2\text{O}_7$ K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>Relaxational Dynamics with  $\tau$ 

$$\tau^{-1} = 2J_{\text{ex}} = 0.17 \text{ meV}$$

# Hierarchy of scales



- $J_K/E_F \ll 1$ :  
perturbation theory on  $J_K$
- $J_{\text{ex}}/E_F \ll 1$ : "Migdal's thm",  
theoretically accessible
- $\lambda = J_K^2/J_{\text{ex}}E_F < 1$ : mean-field  
theory is "exact"

# Mean-field theory on the effective model

# Dominant Pairing Channel

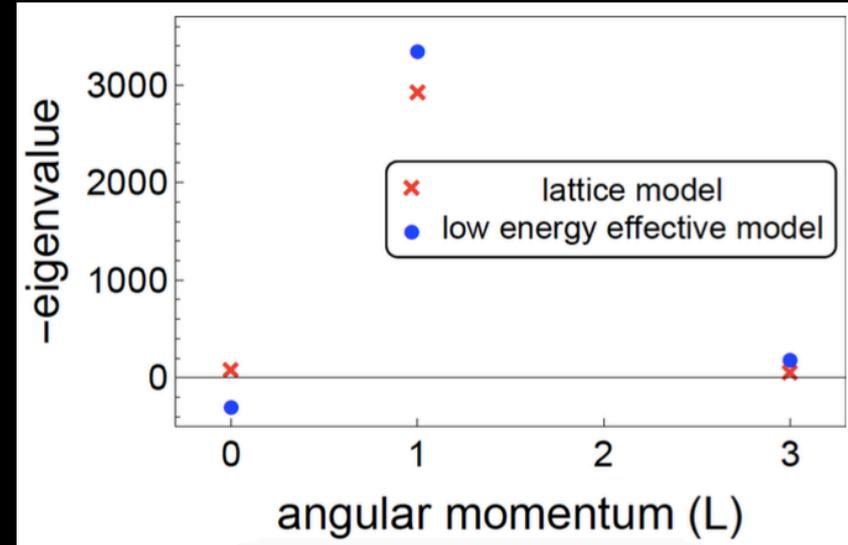
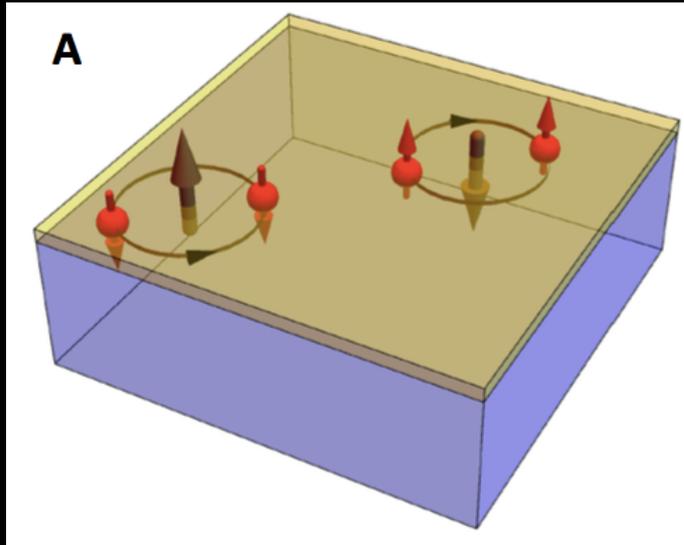
- Key properties of the static spin structure factor

$$S_{ab}(\mathbf{q}) = \delta_{ab} - \left(1 - \frac{1}{1+q^2\xi^2}\right) \frac{q_a q_b}{q^2}$$

1. "spin-orbit" coupling
2.  $J_z = L_z + S_z$  conserved.
3. spin "mirror" symm:  $S_{ab}(\mathbf{q}) = S_{ba}(\mathbf{q})$   
-> singlet - triplet decoupled.

- Purely repulsive interaction in the singlet channel

# Dominant Pairing Channel



1.  $^3\text{He-B}$  type but 2D.
2. Overwhelmingly dominant.

$$T_c$$

In analogy to phonon mediated BCS theory,

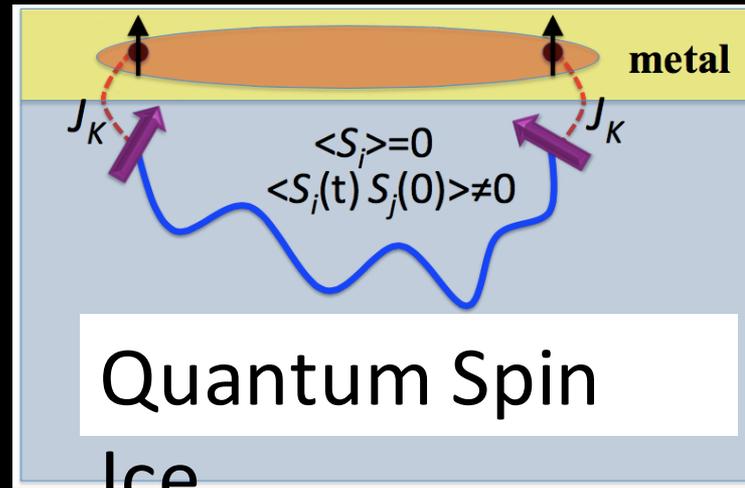
$$T_c \approx \tau^{-1} e^{-1/\lambda}$$

- $\tau^{-1} = 2J_{\text{ex}} = 0.17 \text{ meV}$
- $\lambda = V_{\text{eff}}/E_F = J_K^2/J_{\text{ex}}E_F$

Not bad for a topological superconductor

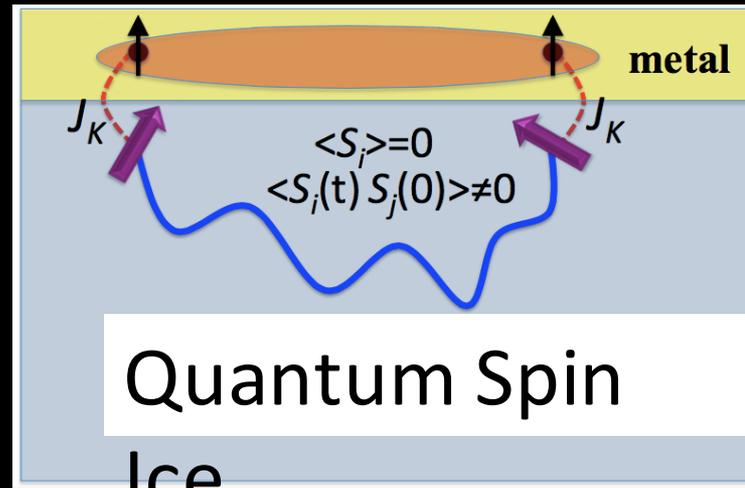
# Microscopic Proposal

# Structural Criteria for the Metal



1. Chemical stability
2. Lattice matching:  $A_2B_2O_7$
3. No orphan bonds: (111) direction

# Electronic Criteria for the Metal



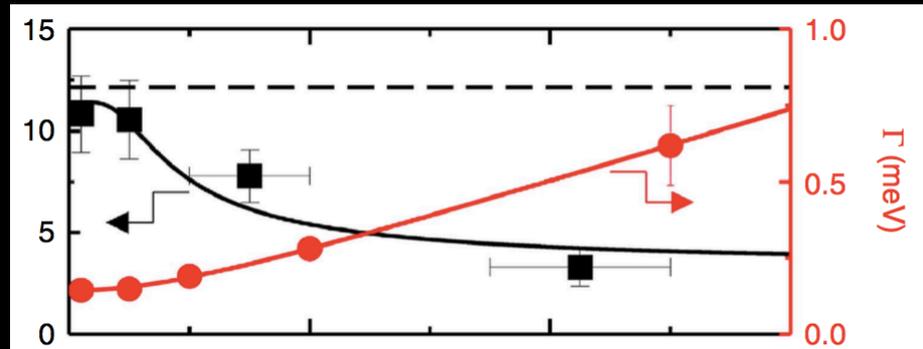
1. Simple metal without ordering possibilities.
2. Wave function penetration for coupling.
3. Odd # of Fermi surface around high symmetry points for a **Topo SC**.

# Transition Temperature

- Spin dynamics

$$T_c \sim \tau^{-1} e^{-1/\lambda}$$
$$\lambda \sim J_K^2 / (\bar{E}_F J_{\text{ex}})$$

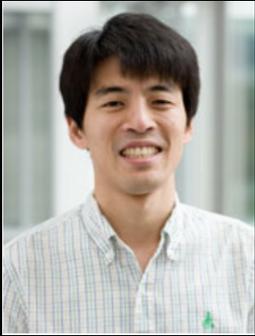
$$\tau^{-1} \sim 2J_{\text{ex}} \sim 0.17 \text{ meV}$$



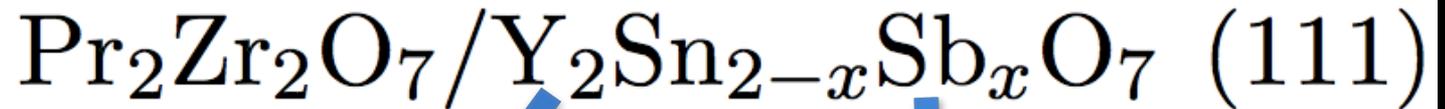
- Parameters for our proposal

$$E_F \sim 300 \text{ meV}, \quad J_K \sim 10 \text{ meV}, \quad \lambda \sim O(1)$$

- $T_c \sim 1.5 \text{ K}$

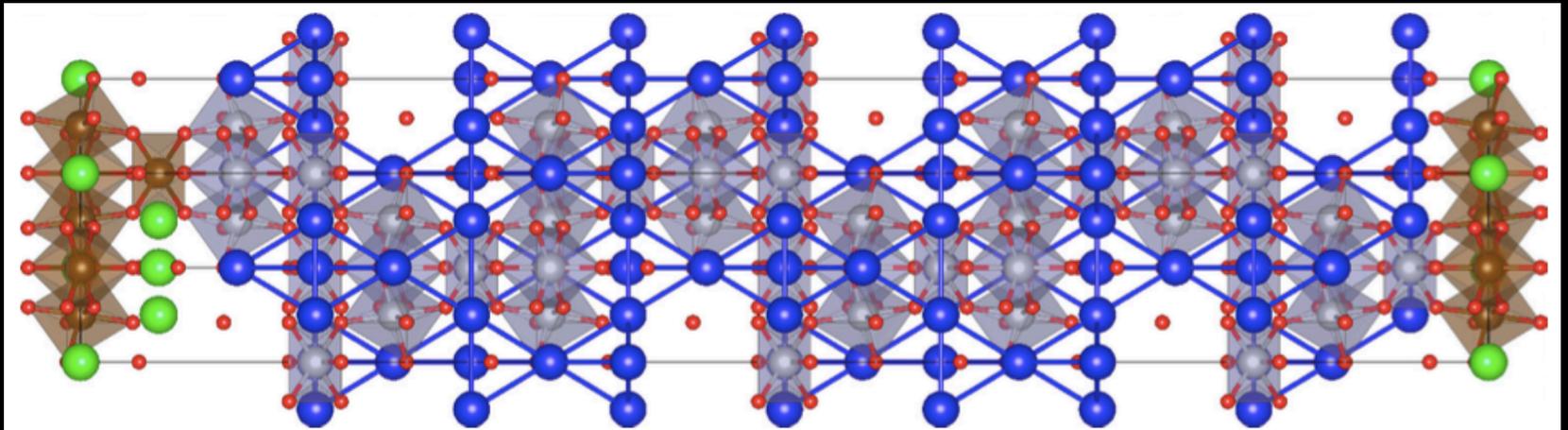


# Microscopic Proposal



Non-magnetic

s-electrons:  
large overlap,  
isotropic FS.

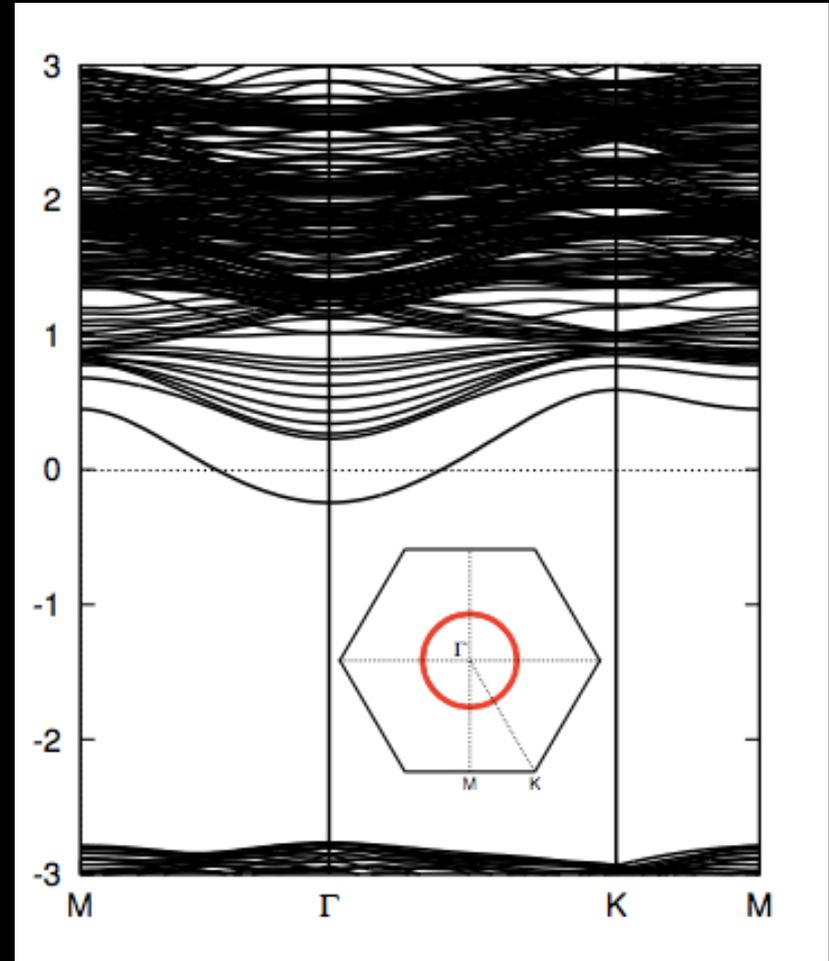


# Band structure for the Proposal

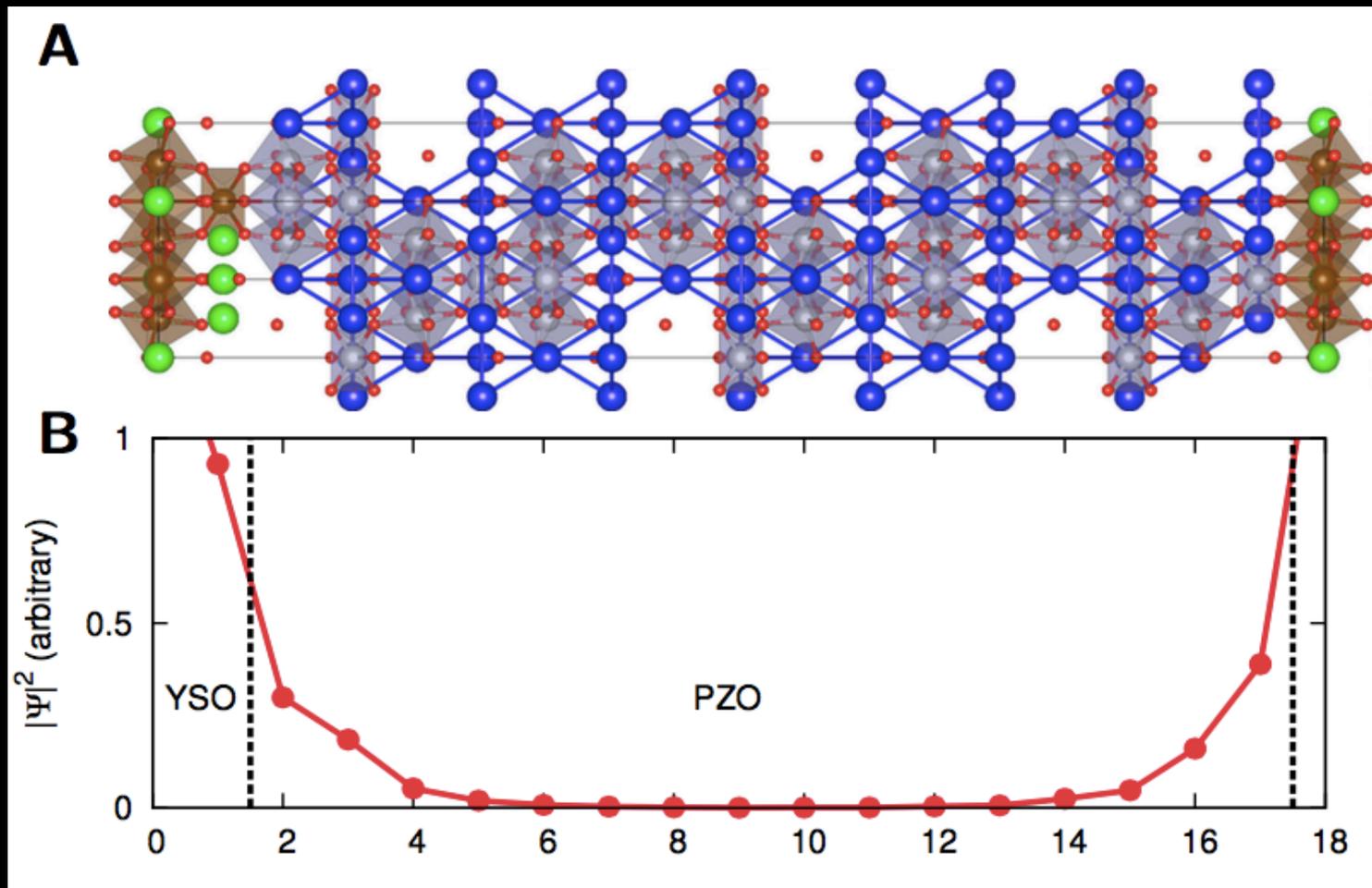
$\text{Pr}_2\text{Zr}_2\text{O}_7/\text{Y}_2\text{Sn}_{2-x}\text{Sb}_x\text{O}_7$  (111)

$x=0.2$

- Isotropic single pocket centered at  $\Gamma$ -point

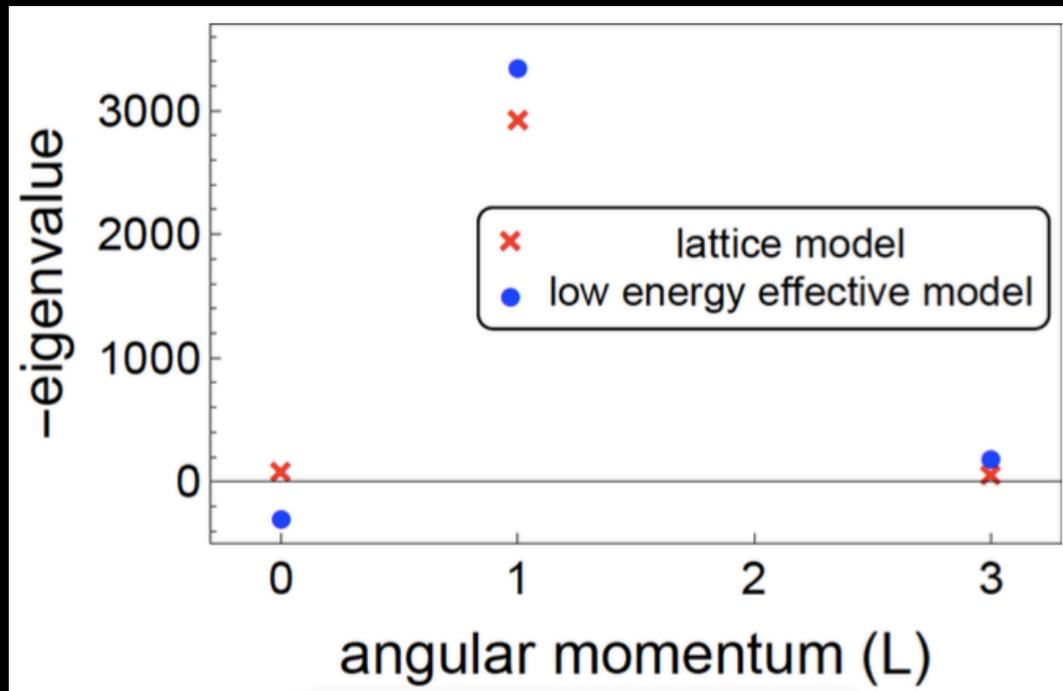


# Wave function penetration



# Full Lattice Model for the proposal

- Effective Continuum theory is valid.
- Ferromagnetic fluctuation is dominant.
- Overwhelmingly dominant p-wave instability.



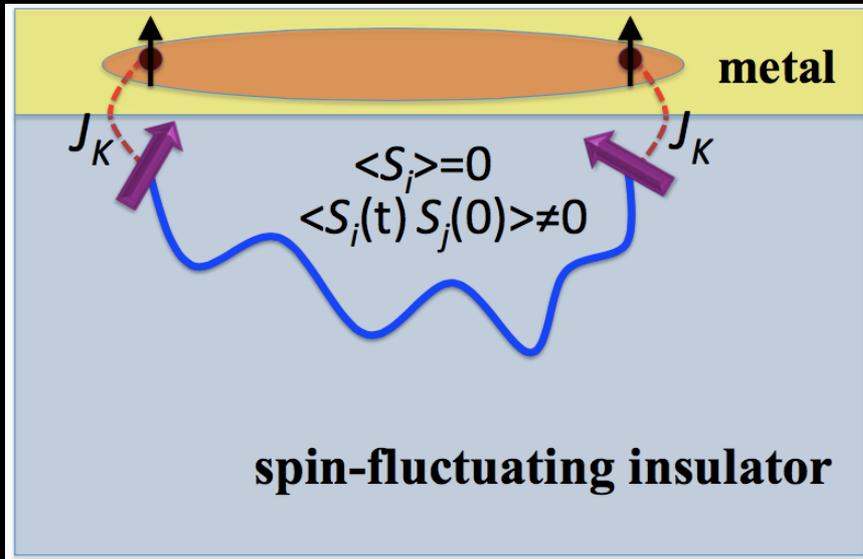
# Earlier Proposal: Excitonic mechanism

- Little (64), Ginzburg (70), Bardeen (73)



- Unstable against exchange.
- Intrinsically s-wave.

# Topological Superconductivity in Metal/Quantum-Spin-Ice Heterostructures

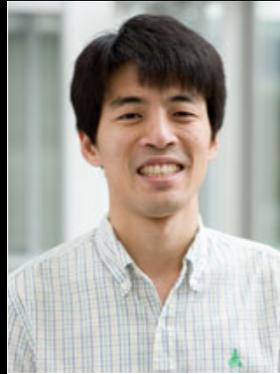


- A new strategy for exploiting spin entanglement of QSL.
- Non-trivial, but tame.
- First T-inv Topo SC.
- Huge phase space.

# Acknowledgements



Jian-huang She



Choonghyun Kim



Criag Fennie



Michael Lawler

# Funding

