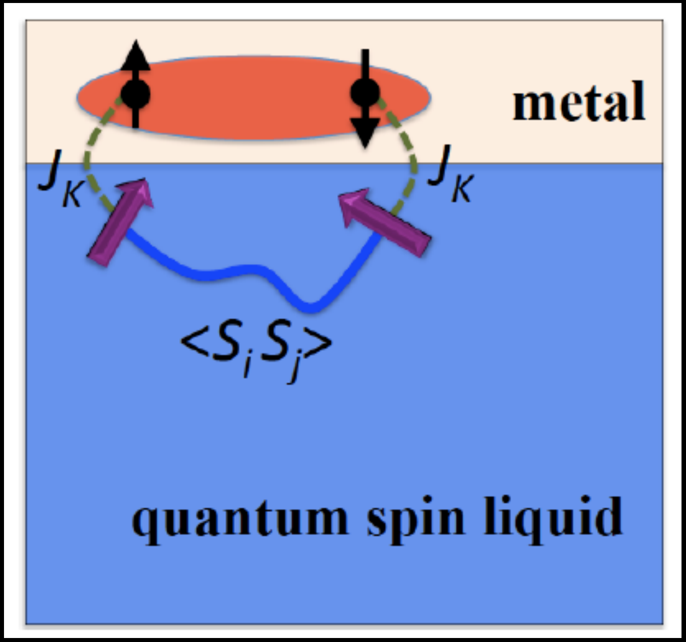
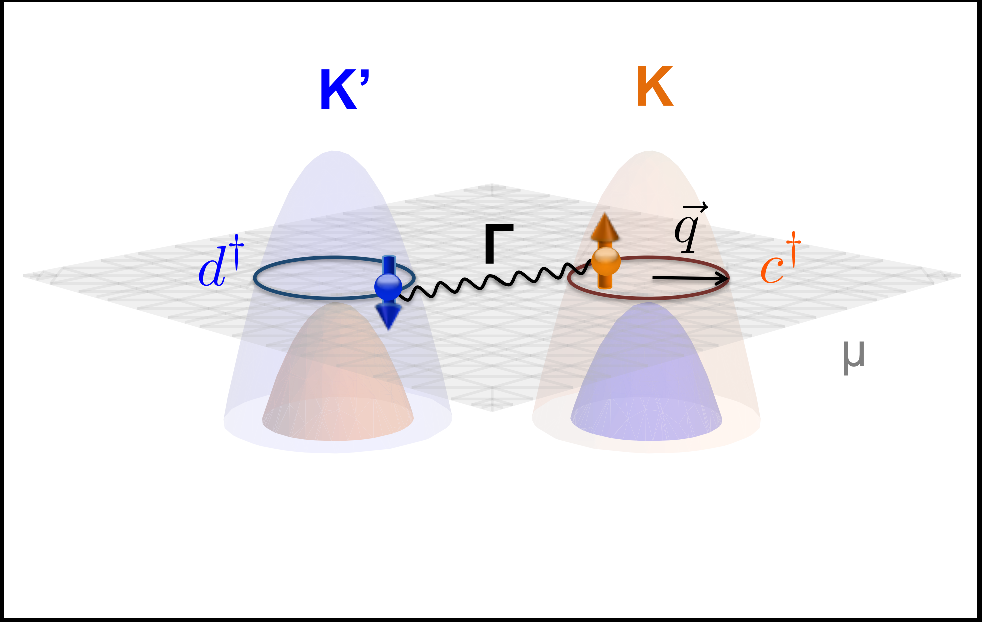
Let There Be Topological Superconductors





Eun-Ah Kim (Cornell)

UIUC 3.28.2016

Q. Topological Superconductor material?

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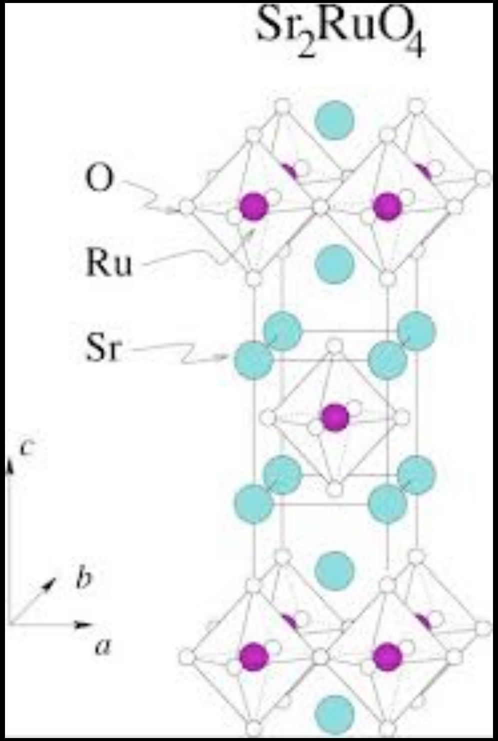
Bulk



Q. Topological Superconductor material?

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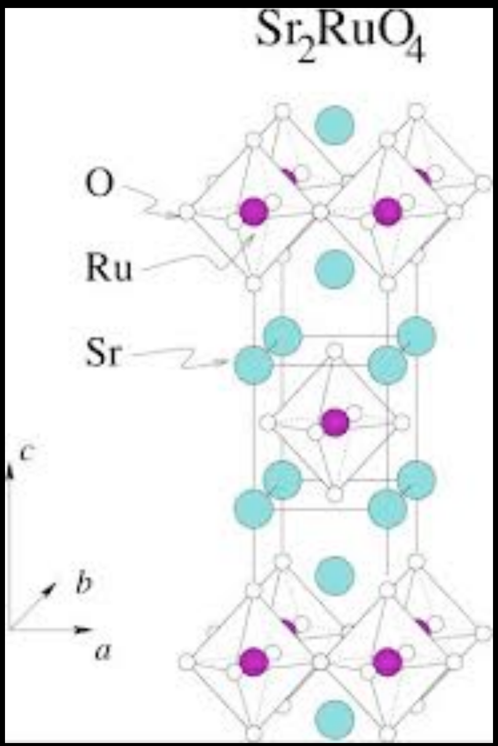
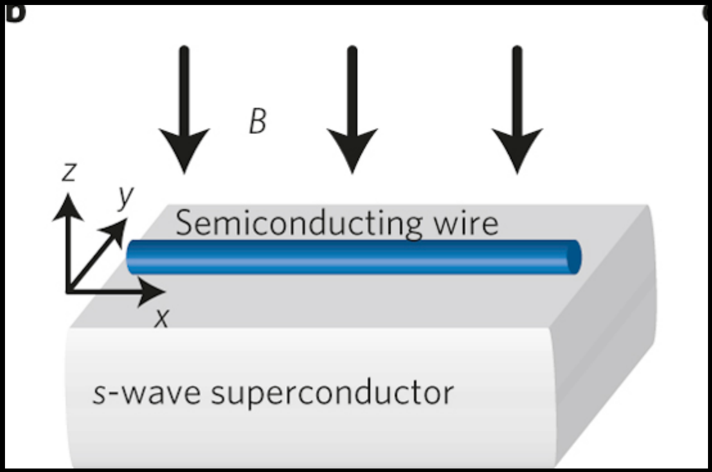


1D proximity



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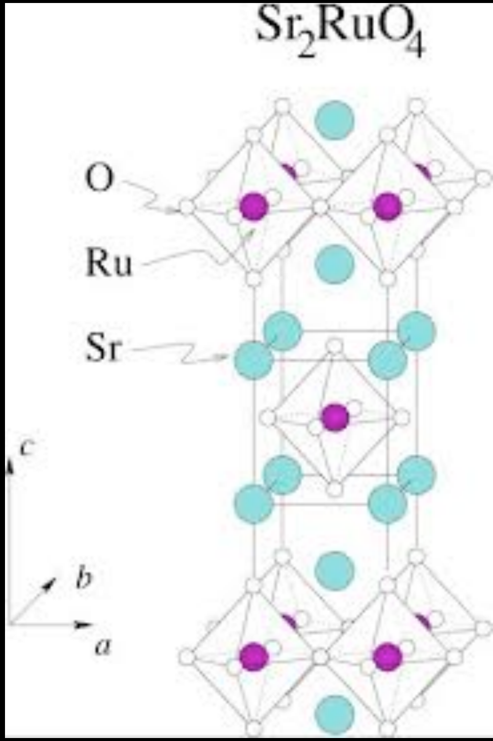

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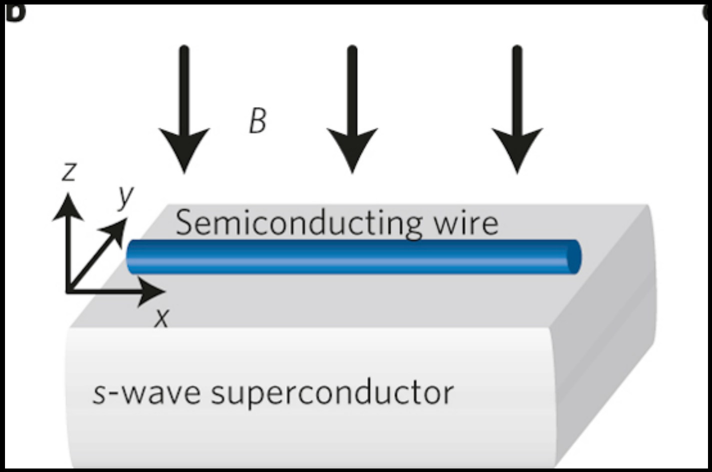
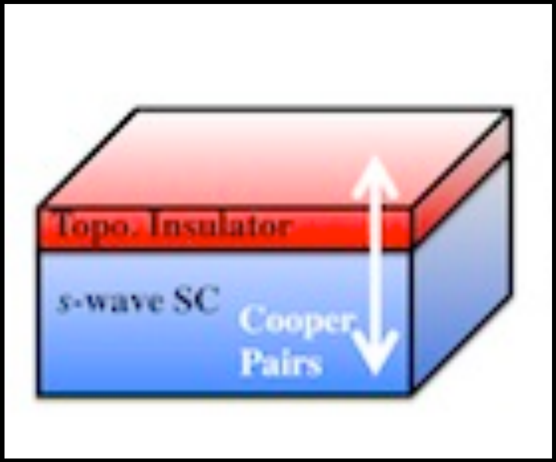


2D proximity?



Designing 2D topological SC's

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- 2D topological SC
  - odd-parity SC of spinless fermions
  - Majorana bound state

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- 2D topological SC
  - odd-parity SC of spinless fermions
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- Strategies:

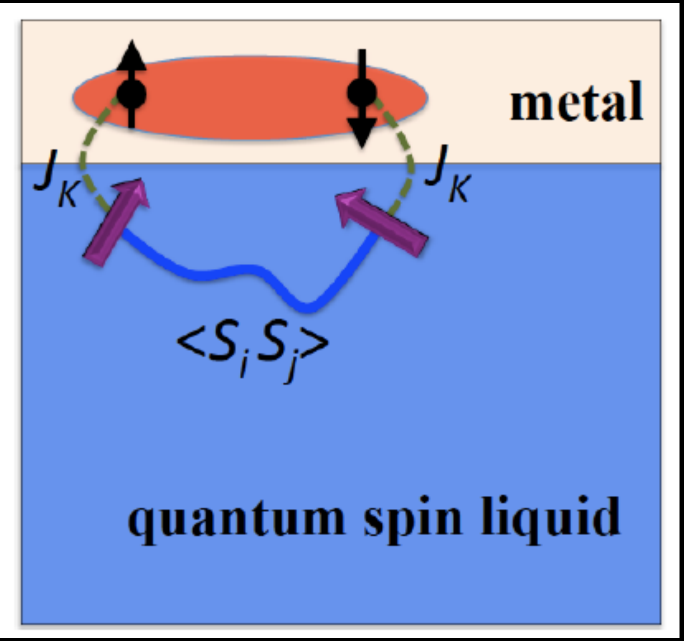
1) interaction,

2) spinlessness

Strategy I

- Manipulate the pairing interaction: target non-phononic mechanism

Topological Superconductivity in Metal/ Quantum-Spin-Ice Heterostructures



Jian-Huang She, Choonghyun Kim, Craig Fennie,
Michael Lawler, E-AK (arXiv:1603.02692)

Wanted: non-phononic mechanism

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P.W.Anderson

Dope a Quantum spin liquid

Wanted: non-phononic mechanism

Dope a Quantum spin liquid

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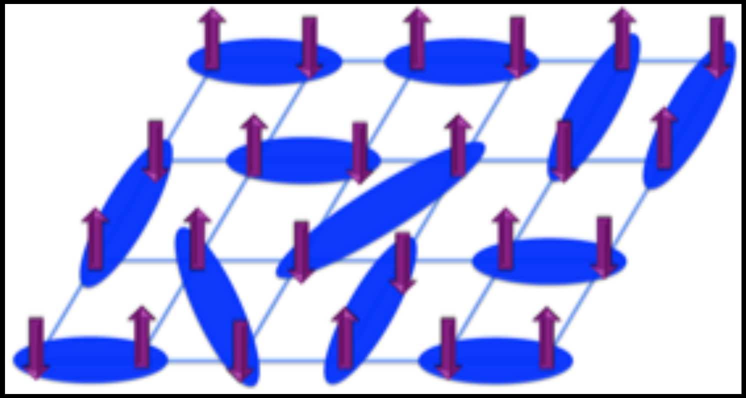

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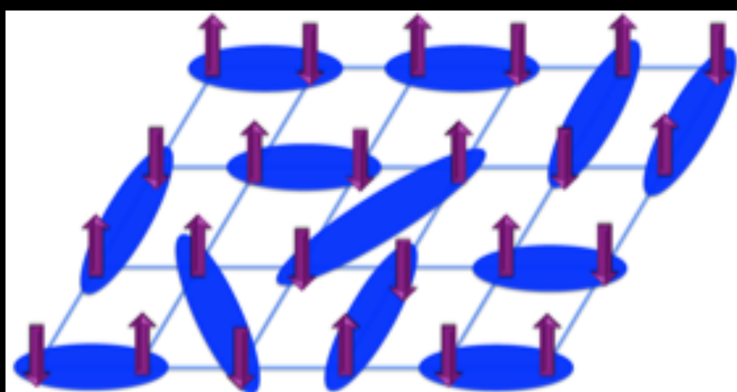

RVB singlet

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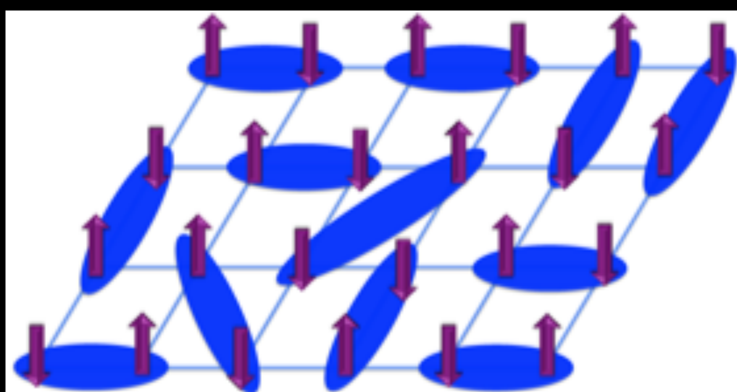
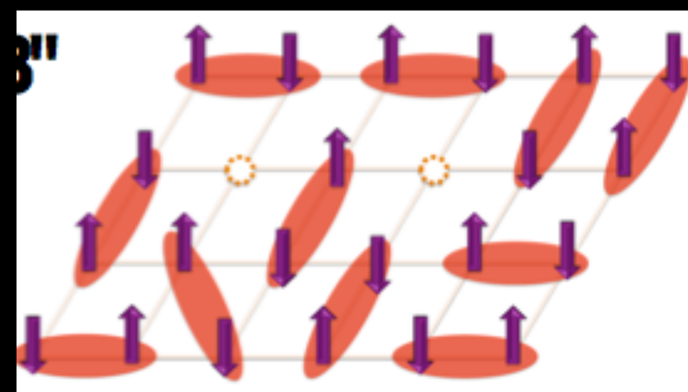
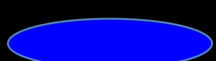
P.W.Anderson







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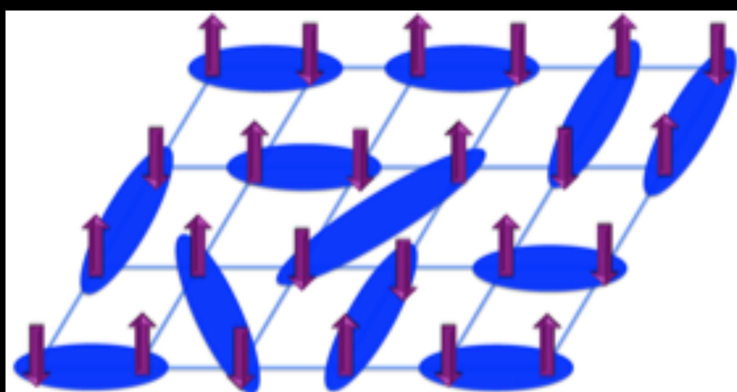
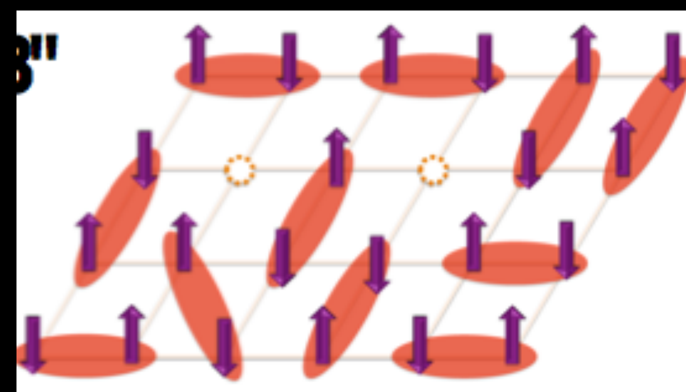


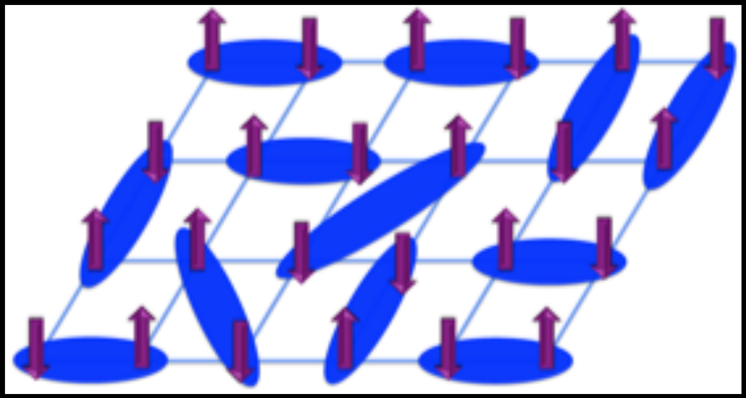


RVB singlet

Cooper pair single

Wanted: non-phononic mechanism

Use Quantum spin liquid



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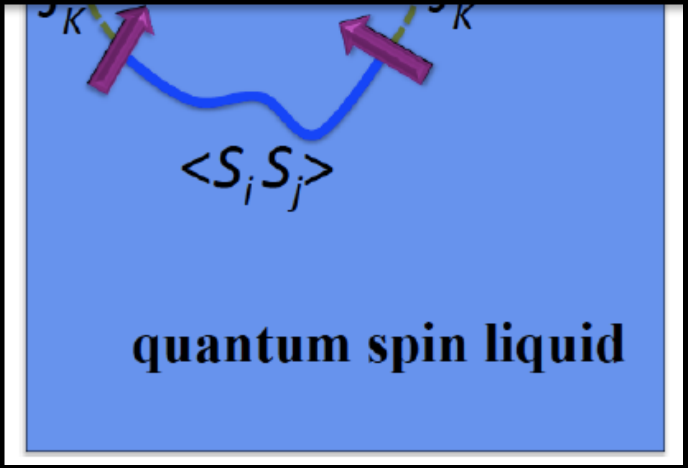

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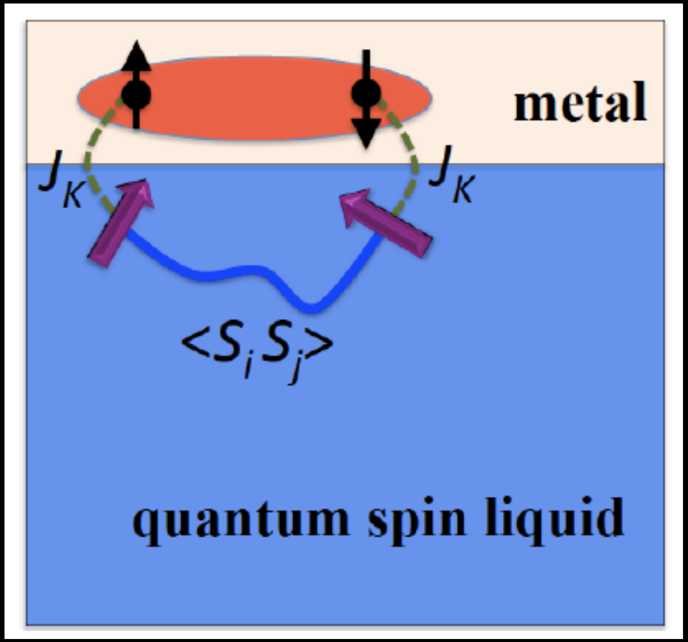


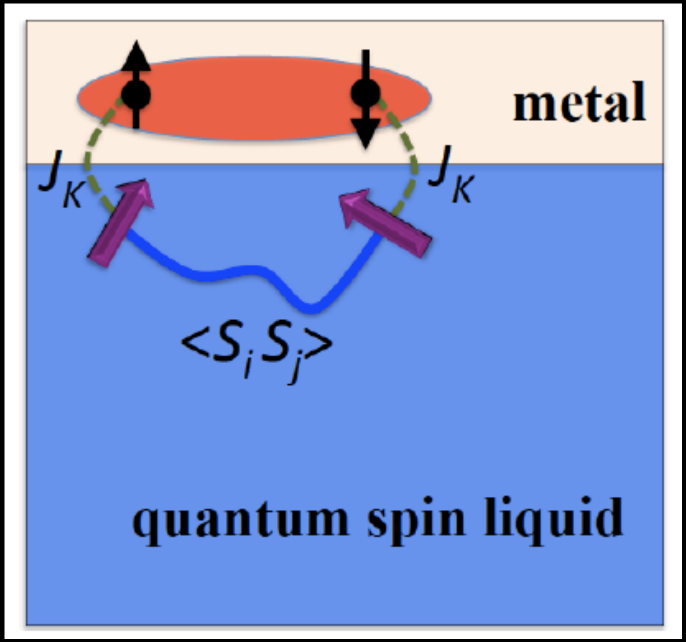
E_F

J_e

x

Wanted: non-phononic mechanism

Use Quantum spin liquid



E_F

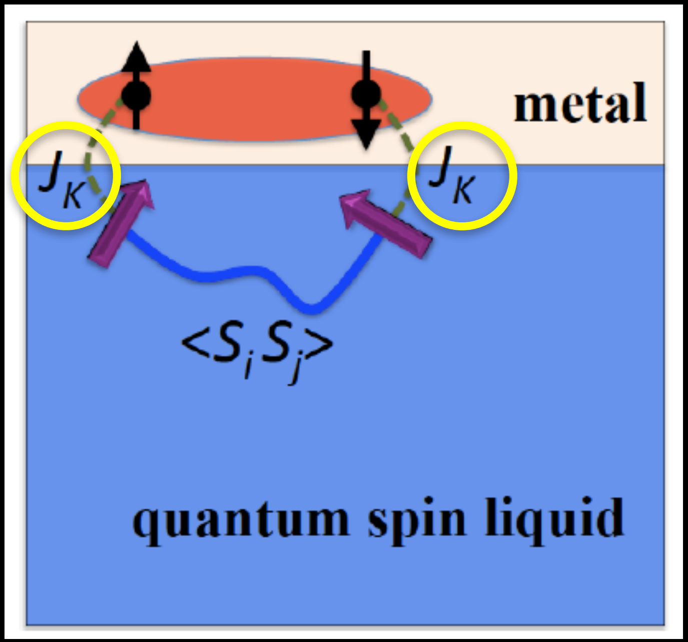
J_e

x

- Characteristic energy scales:

Wanted: non-phononic mechanism

Use Quantum spin liquid



E_F

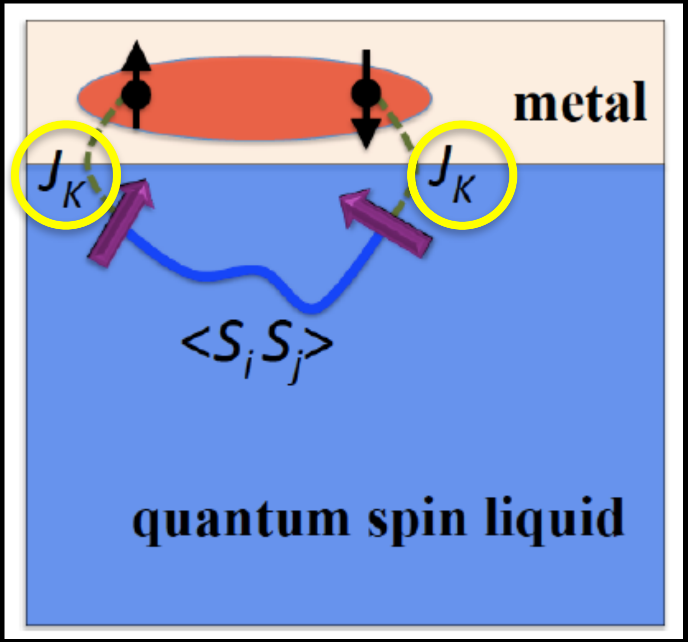
J_{ex}

- Characteristic energy scales:

  E_F, J_{ex}, J_K

Wanted: non-phononic mechanism

Use Quantum spin liquid



E_F

J_{ex}

- Characteristic energy scales:

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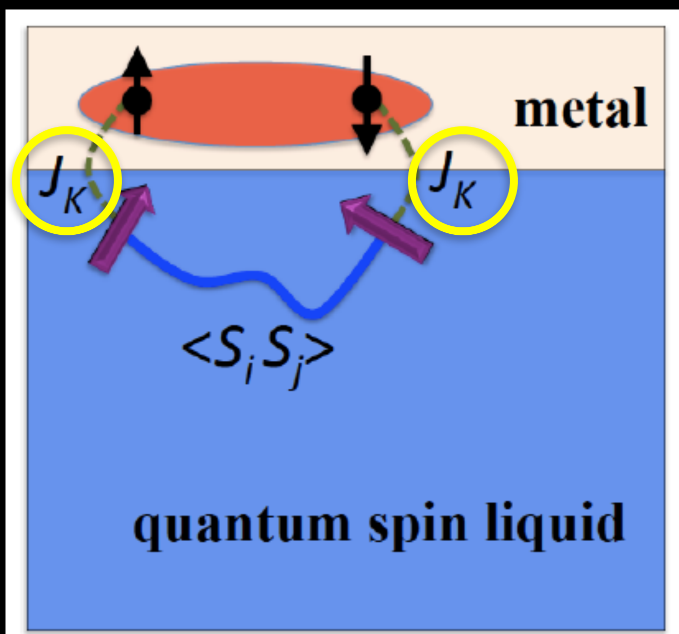
- Perturbative limit:

  $J_K / E_F << 1$

Wanted: non-phononic mechanism

Use Quantum spin liquid



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- Spin-fermion model

Spin-fermion model for J_{ex}=0

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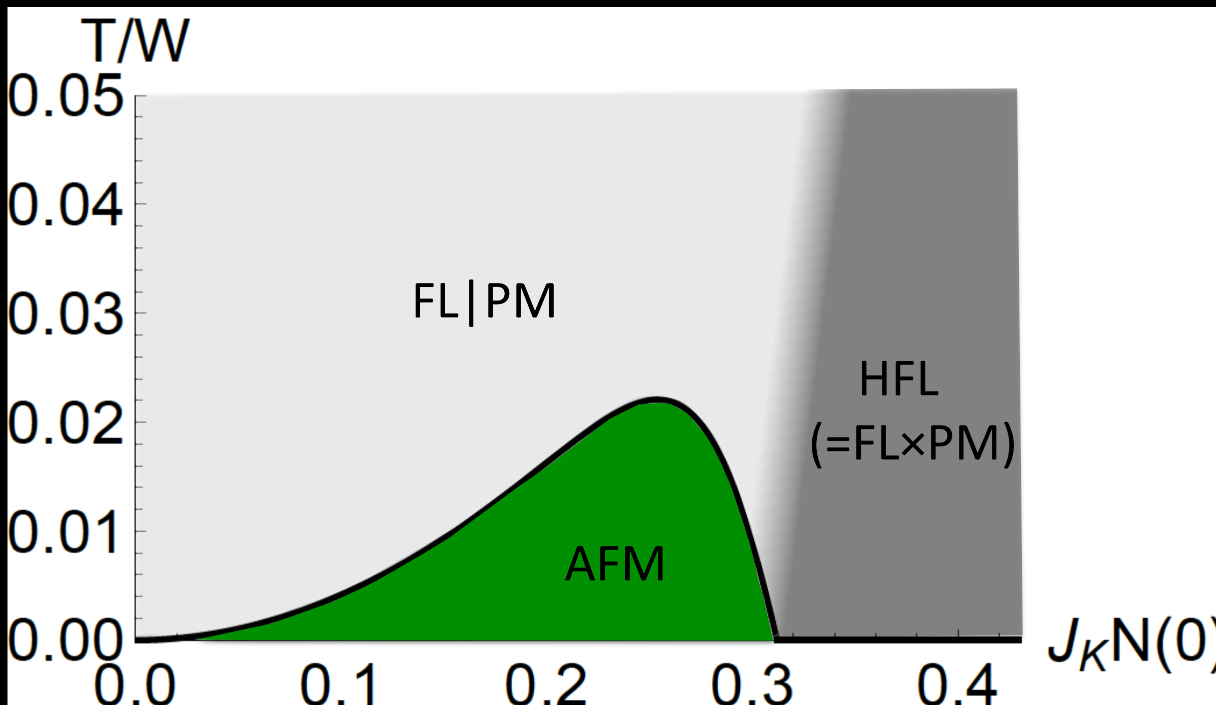
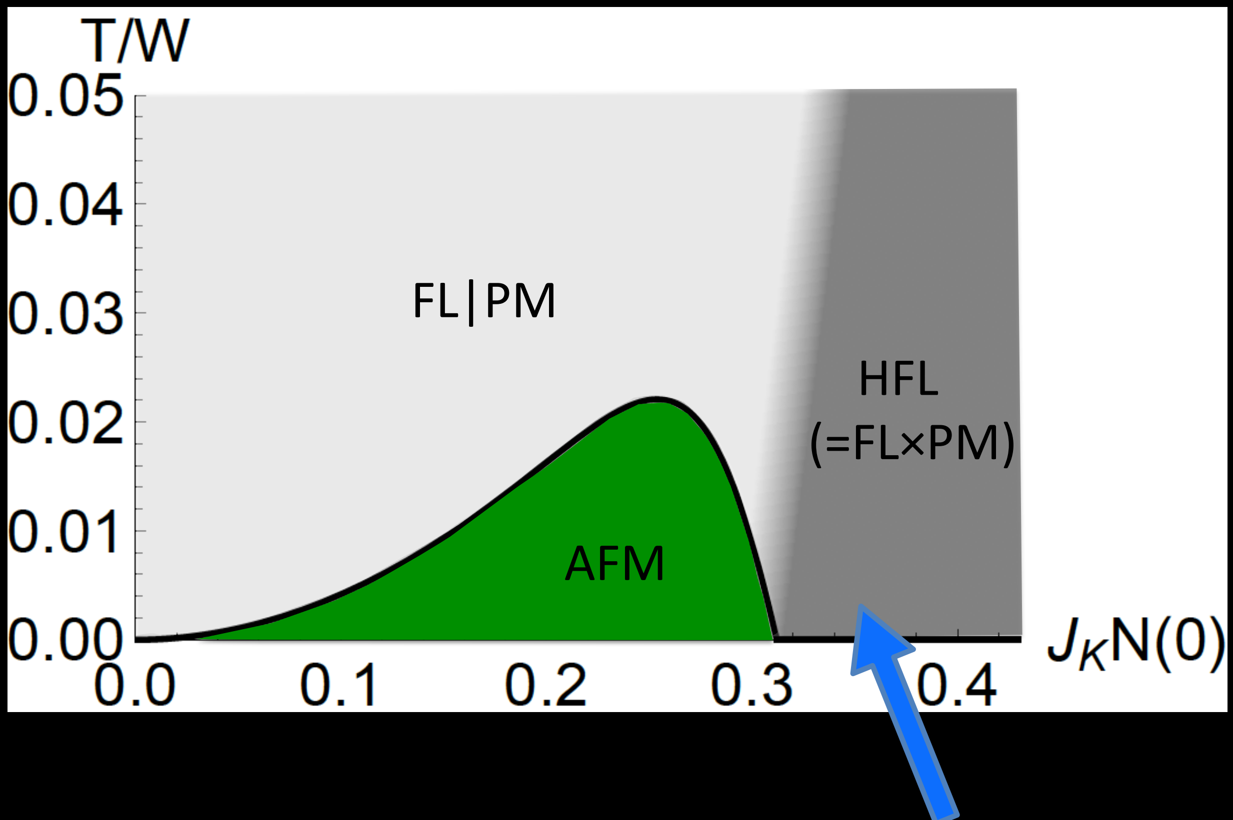


Doniach (1977)

Spin-fermion model for J_{ex}=0



Kondo-Singlet

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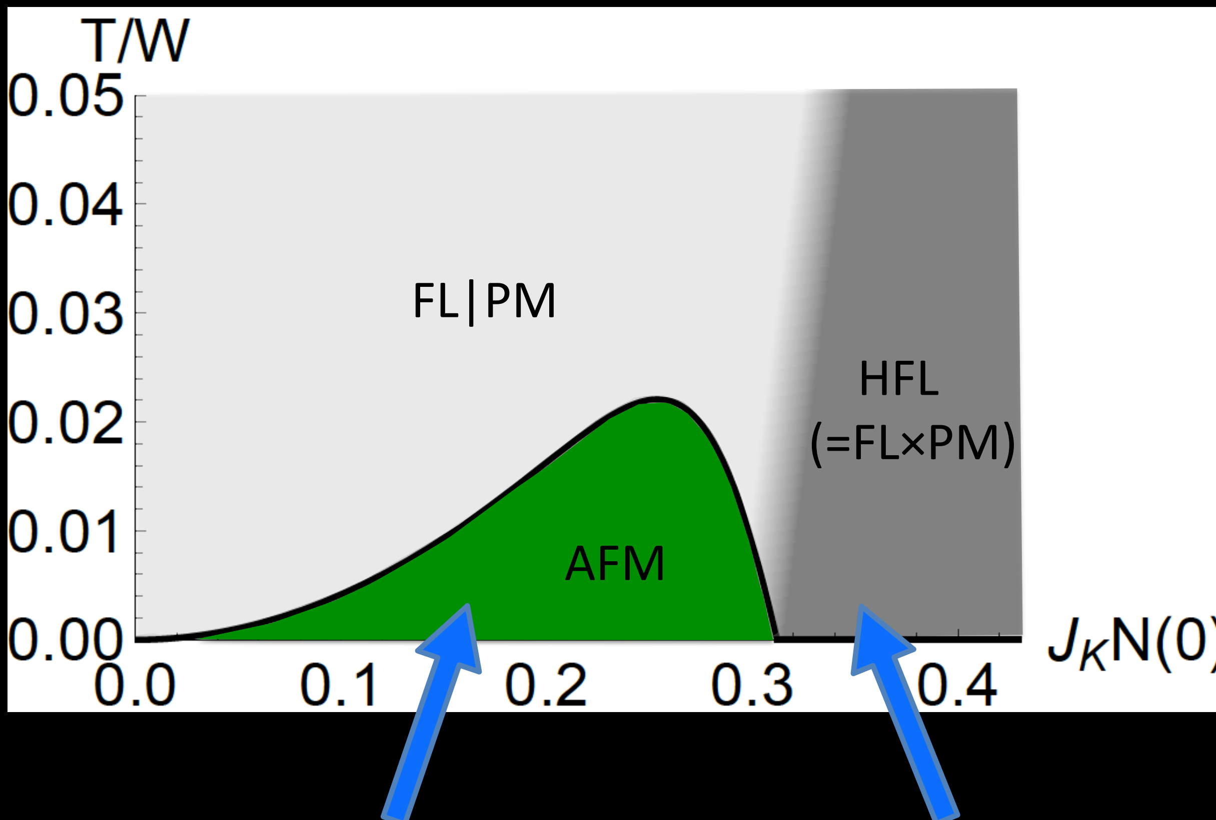

RKKY interaction

Kondo-Singlet

Doniach (1977)

Spin-fermion model for J_{ex}+ Frustration

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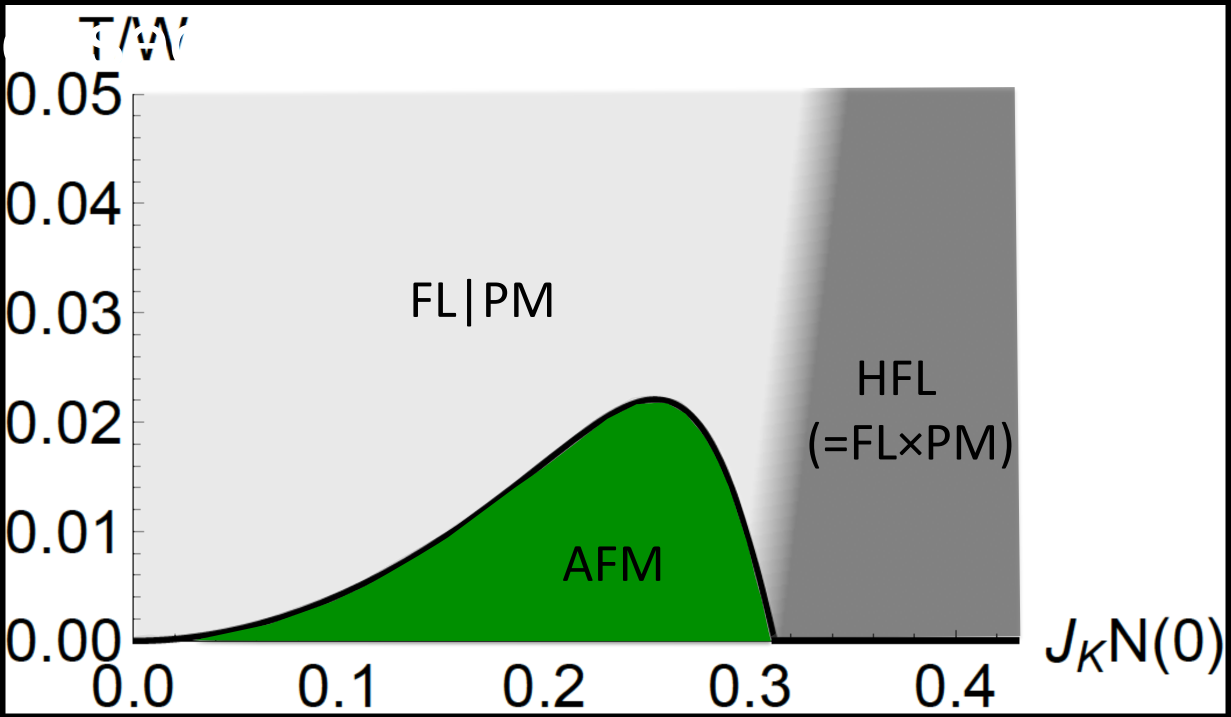
For $J_{RKKY} \sim J_K^2 N(0) < J_{ex}$ AFM order suppressed.

Spin-fermion model for J_{ex} + Frustration

For $J_{RKKY} \sim J_K^2 N(0) < J_{ex}$ AFM order

suppr



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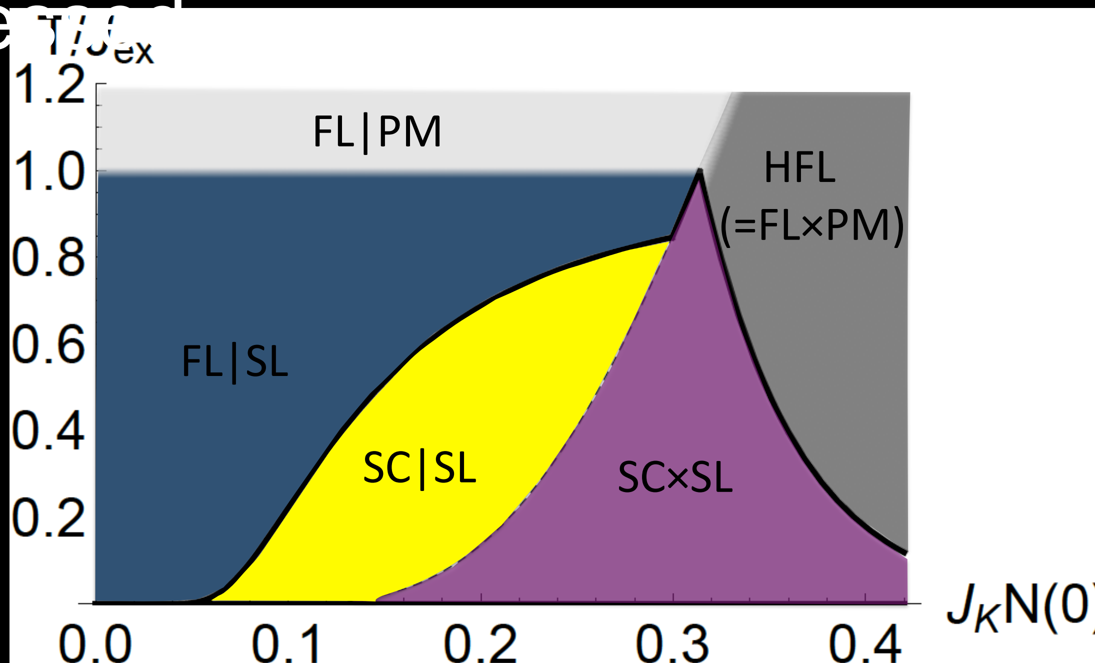

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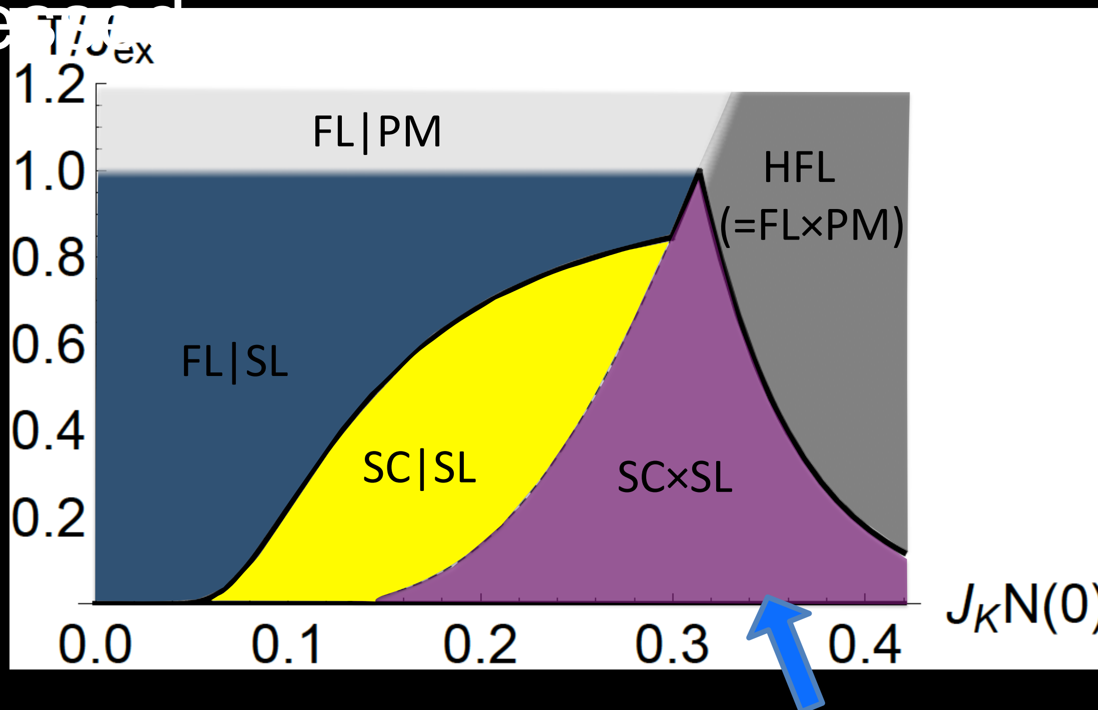

Kondo-Singlet + RVB singlet+Cooper pair singlet

Coleman & Andrei (1989)

Senthil, Vojta, Sachdev (2003)

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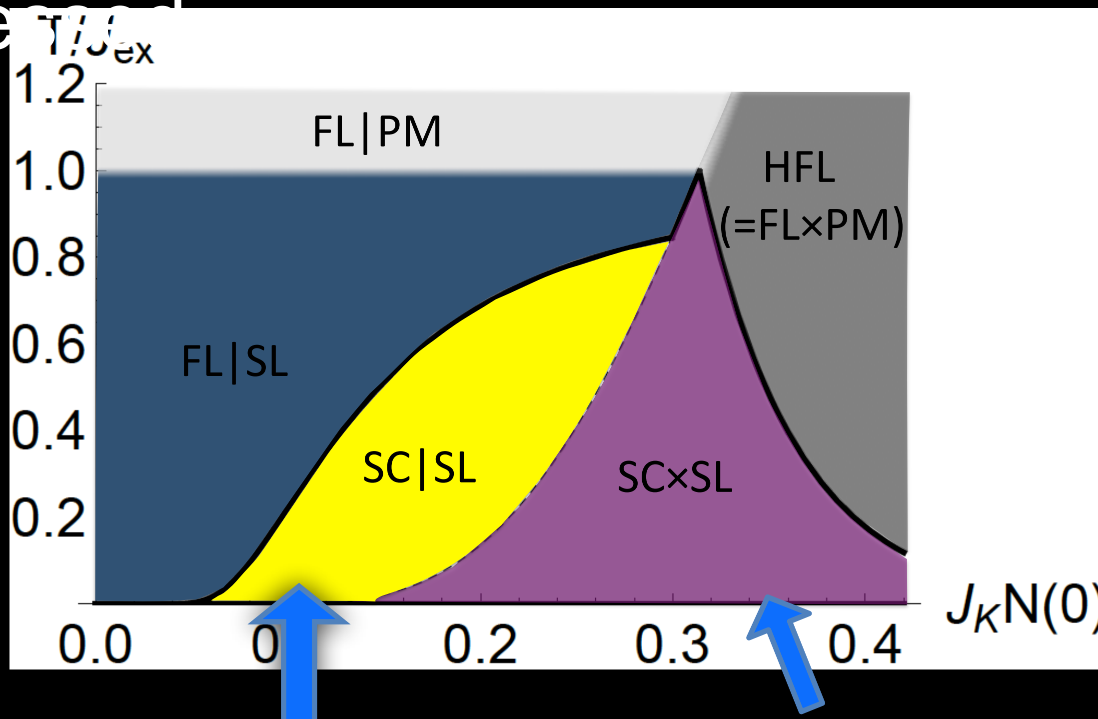

Superconduct or "riding" on QSL

Kondo-Singlet + RVB singlet+Cooper pair singlet

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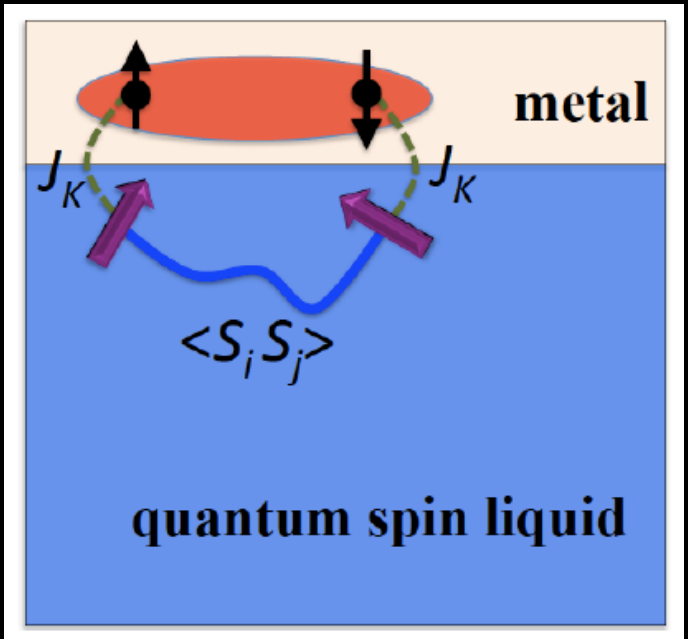
How to predictively materialize SCIQSL ?



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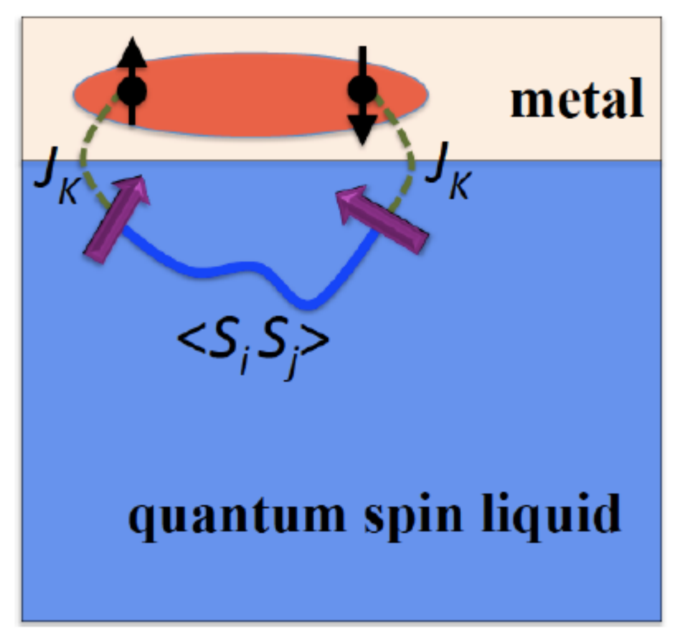

Simple isotropic metal

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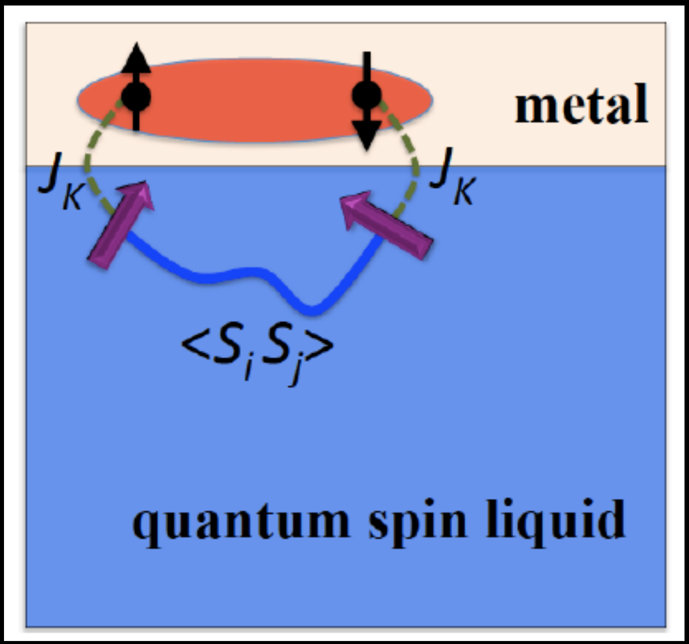

Simple isotropic metal

1. <S>=0

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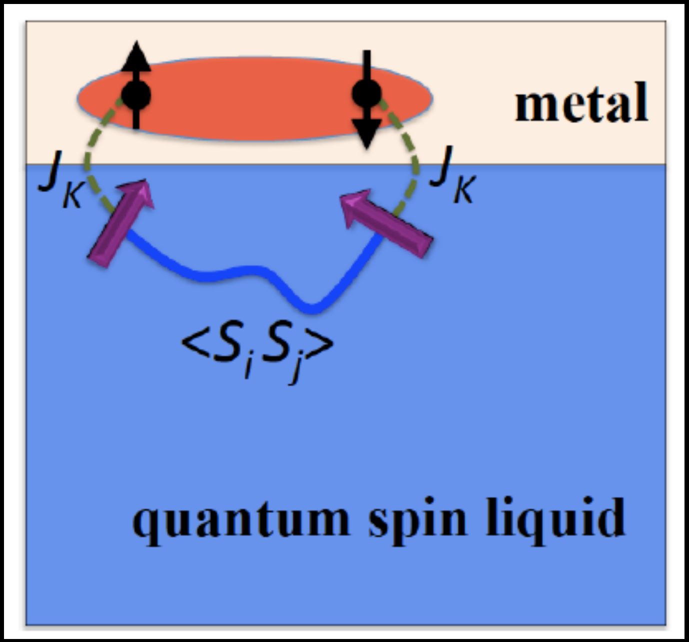
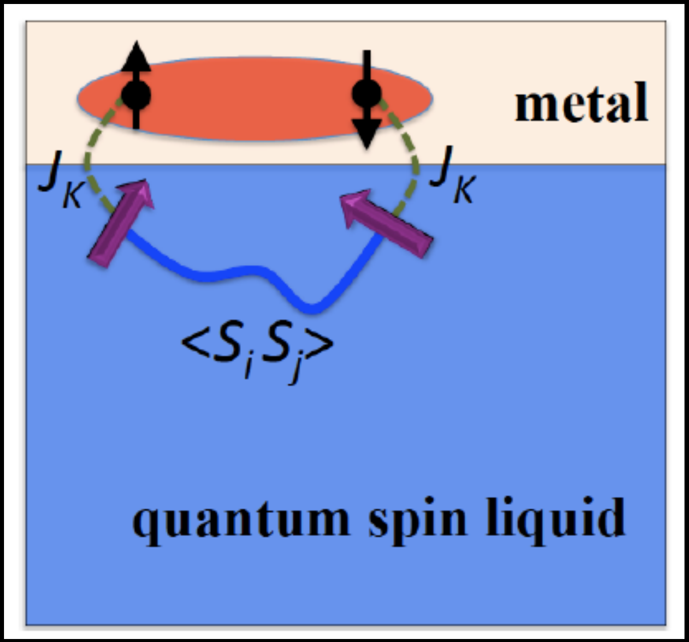


Simple isotropic metal

1. $<S>=0$
2. Dynamic spin fluctuation $<S_i S_j>$

How to predictively materialize SCIQSL ?



Simple isotropic metal

1. **$<S>=0$**
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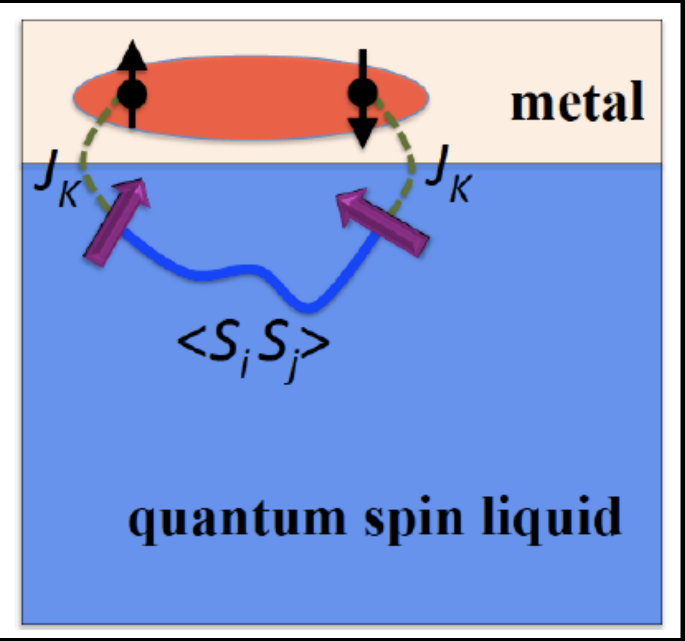
3. **Gapped spectrum**

How to predictively materialize SCIQSL ?



Simple isotropic metal

1. $\langle S \rangle = 0$
2. Dynamic spin fluctuation $\langle S_i S_j \rangle$
3. Gapped spectrum
4. Well understood

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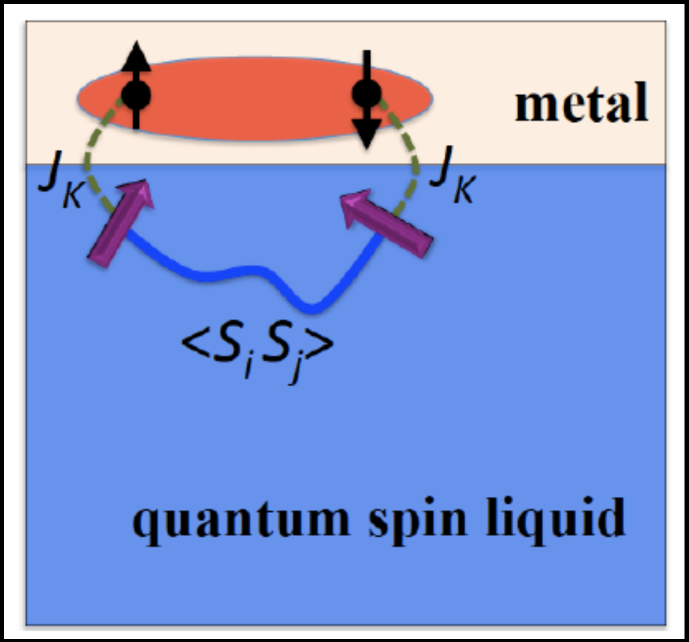

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➡ Quantum Spin Ice

Received 20 Sep 2012 | Accepted 18 Apr 2013 | Published 17 Jun 2013

DOI: 10.1038/ncomms2914

Quantum fluctuations in spin-ice-like $Pr_2Zr_2O_7$

K. Kimura[1], S. Nakatsuji[1,2], J.-J. Wen[3], C. Broholm[3,4,5], M.B. Stone[5], E. Nishibori[6] & H. Sawa[6]

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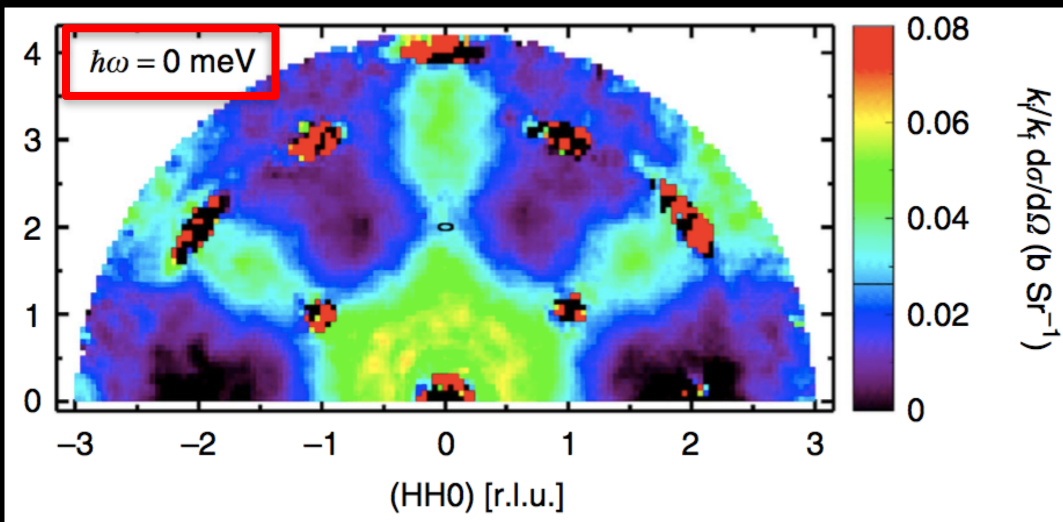

- Elastic neutron: pinch points (spin-ice like)

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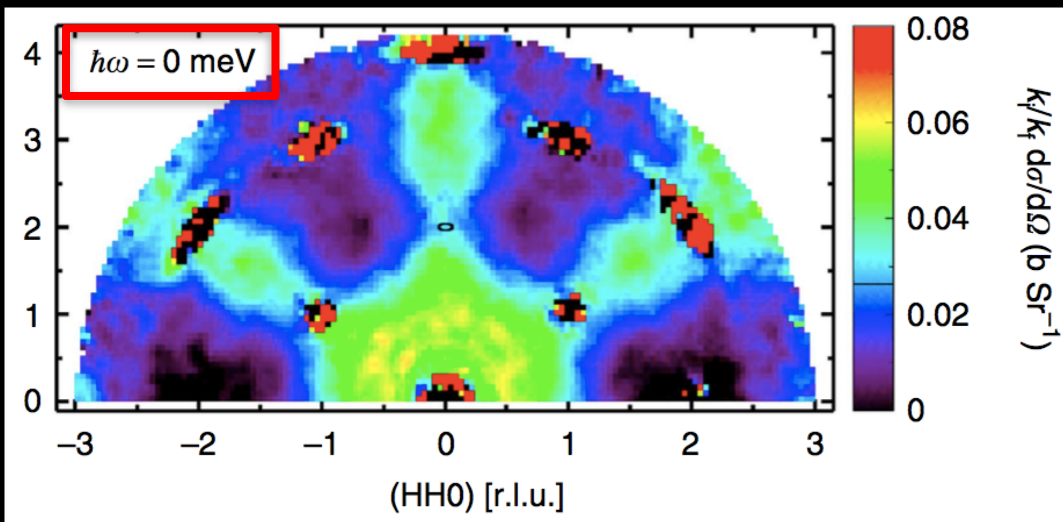

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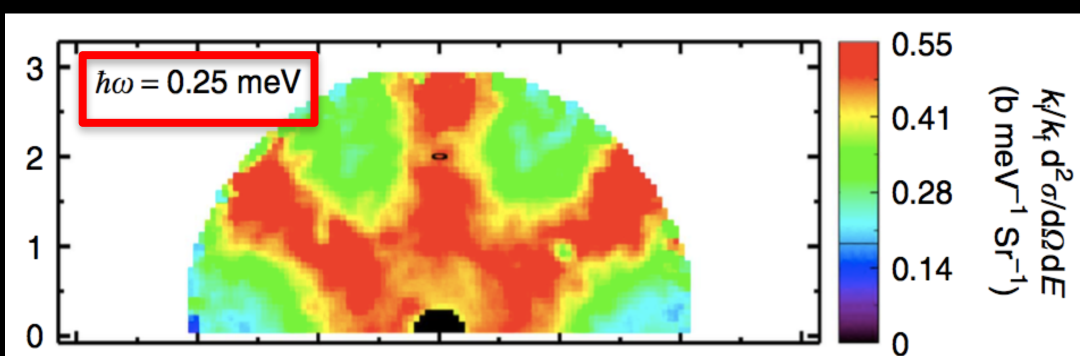

- Inelastic neutron: over 90% weight

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- No order down to 20mK

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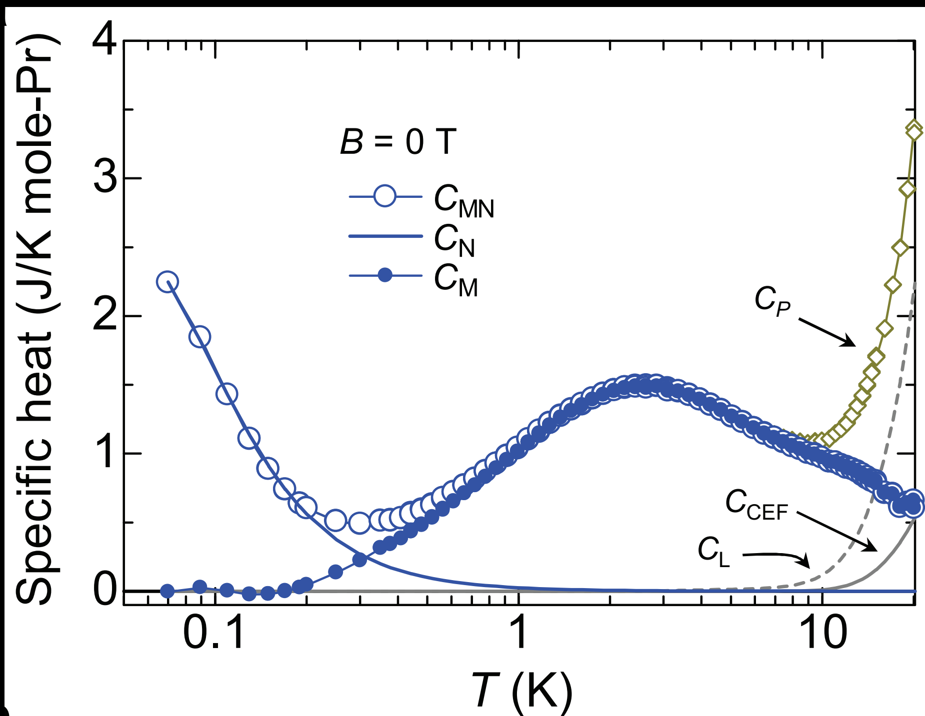

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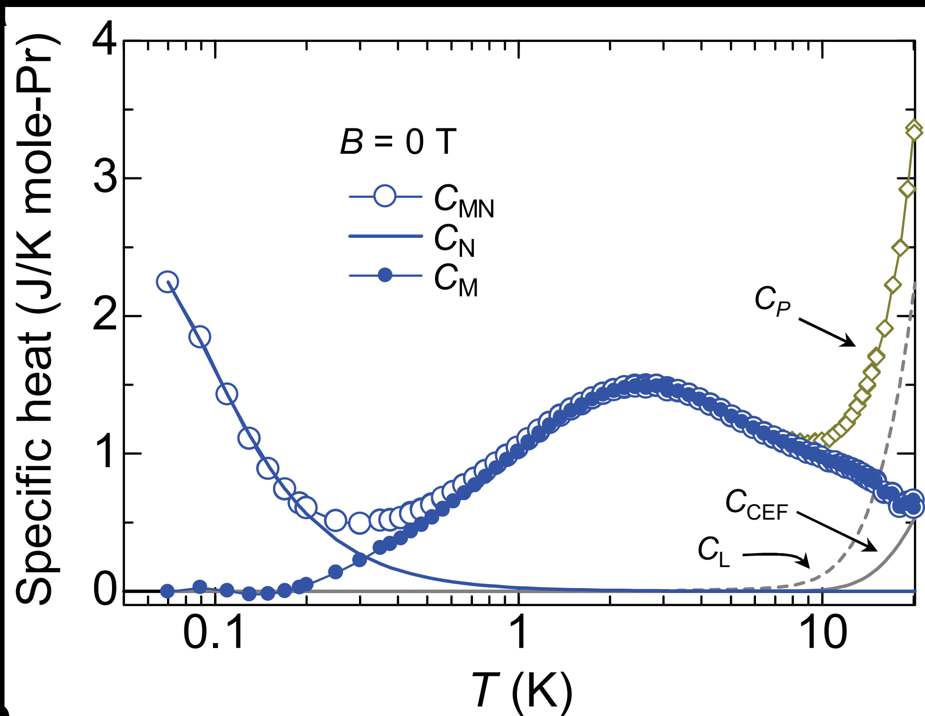

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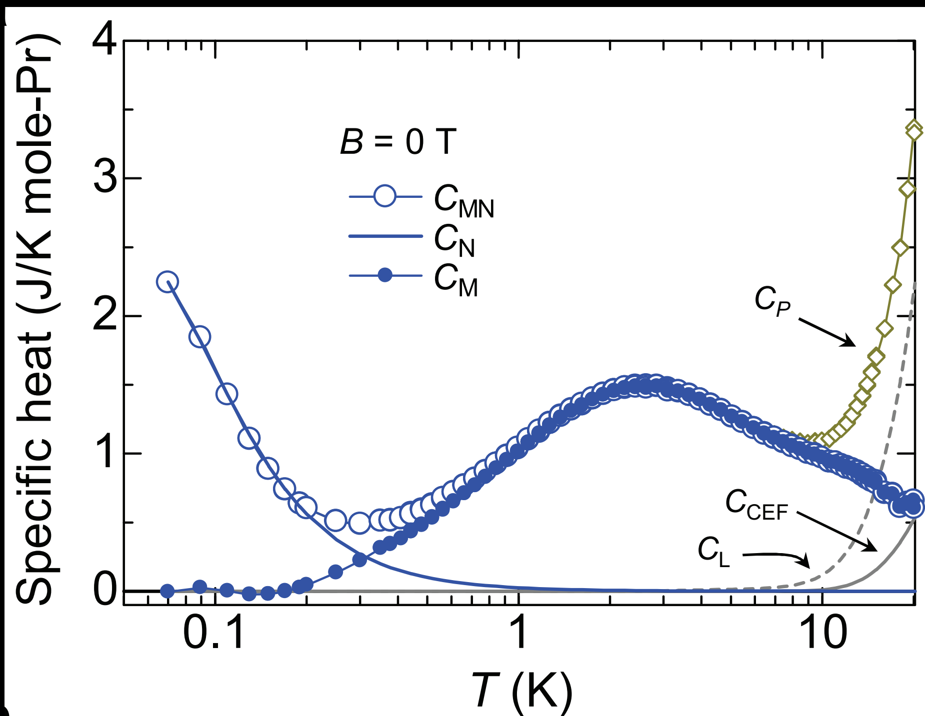

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- Gapped quantum paramagnet ω_s=0.17meV
- Inelastic spectra peaked at Q=0

Emergent Gauge Field in Spin Ice

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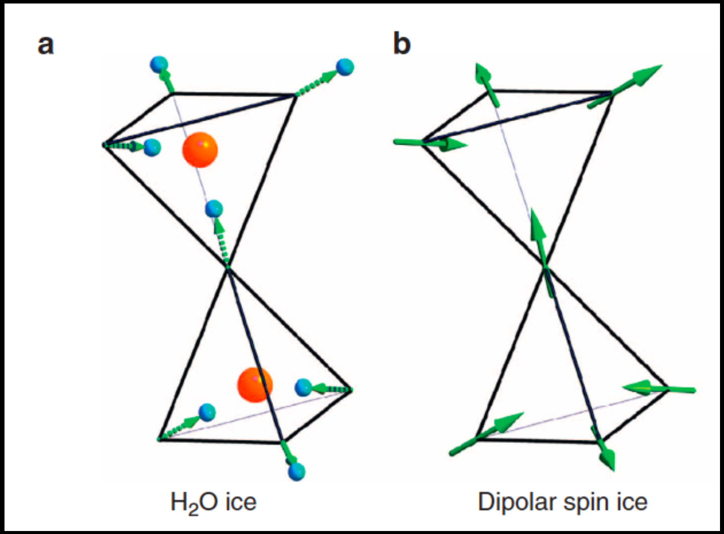

Kimura et al (2013)

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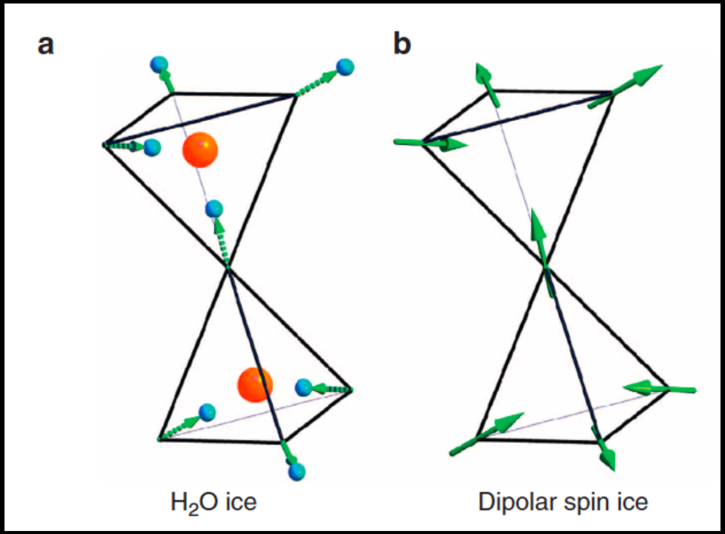
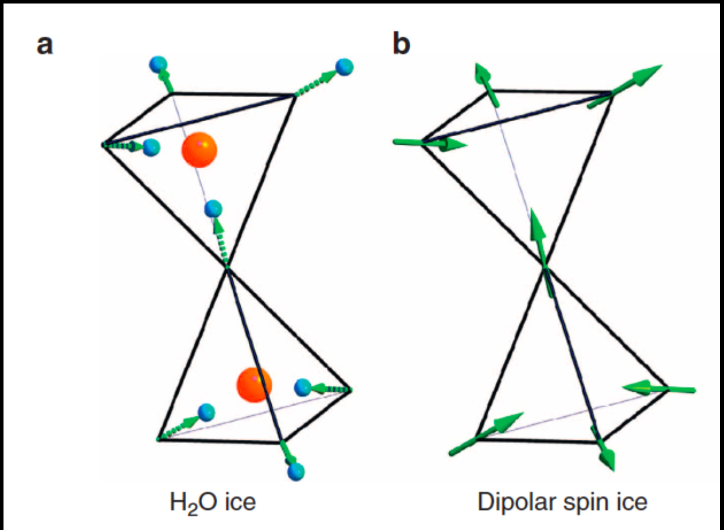


Kimura et al (2013)

- 2-in 2-out ice rule

Emergent Gauge Field in Spin Ice



Kimura et al (2013)

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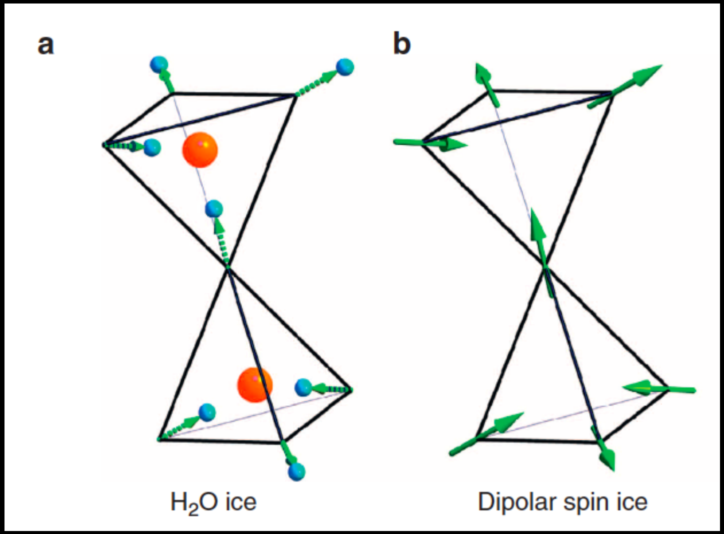
$$\nabla \cdot \vec{S}(\boldsymbol{r}) = 0$$

Emergent Gauge Field in Spin Ice



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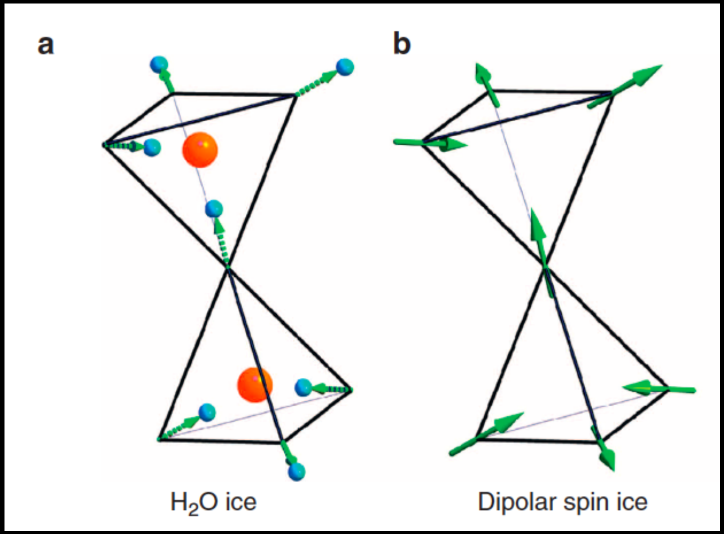

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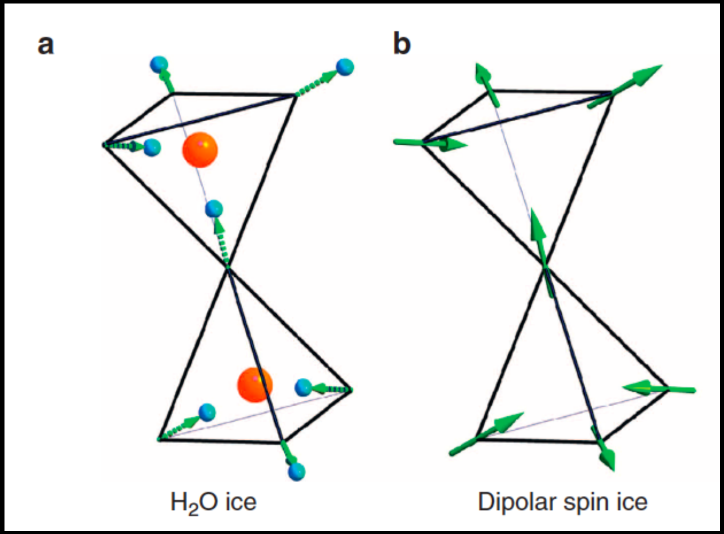
- Gauge Field Propagator

$$\langle A_a(\boldsymbol{q}) A_b(-\boldsymbol{q}) \rangle \sim \frac{1}{q^2} \left(\delta_{ab} - 2\hat{q}_a \hat{q}_b \right)$$

Emergent Gauge Field in Spin Ice



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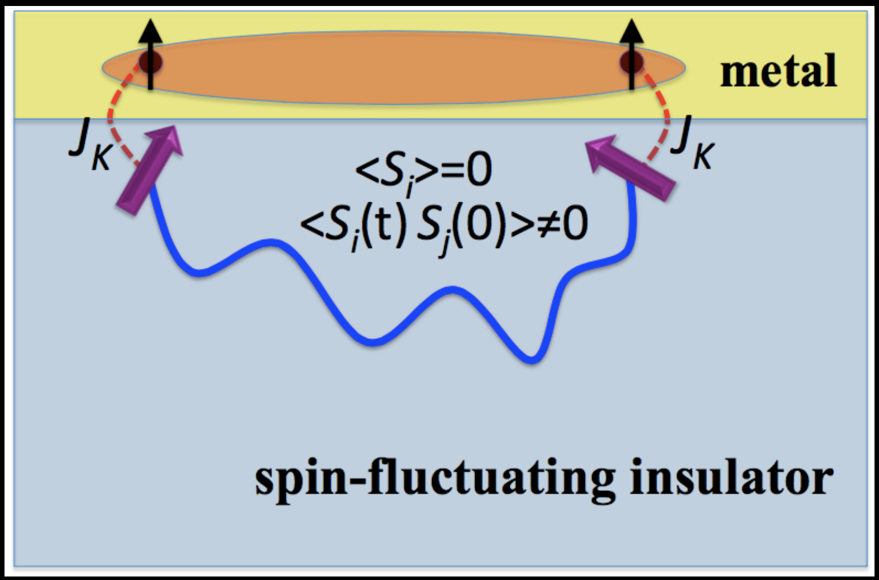
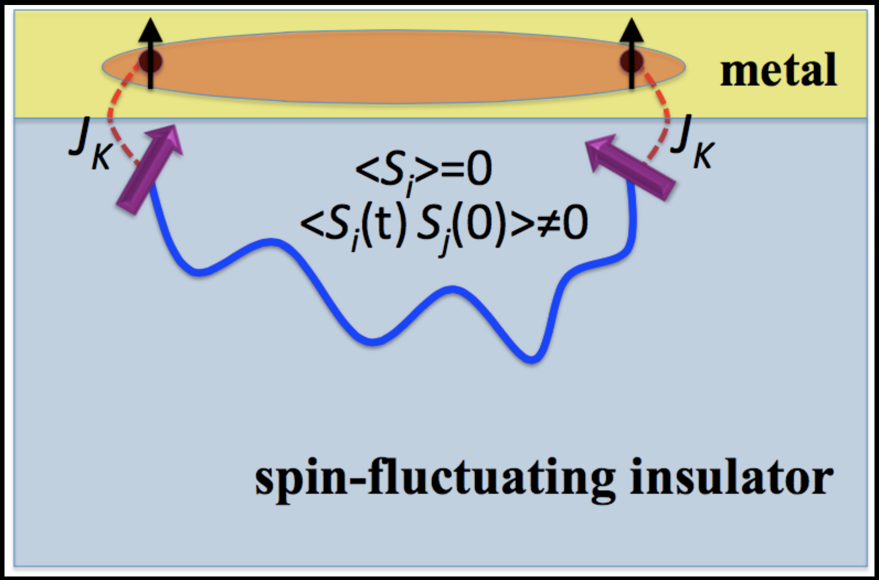
- Spin-spin correlation

$$\langle S_a(\boldsymbol{q}) S_b(-\boldsymbol{q}) \rangle \sim \delta_{ab} - \hat{q}_a \hat{q}_b$$

Effective Continuum Theory



Effective Continuum Theory



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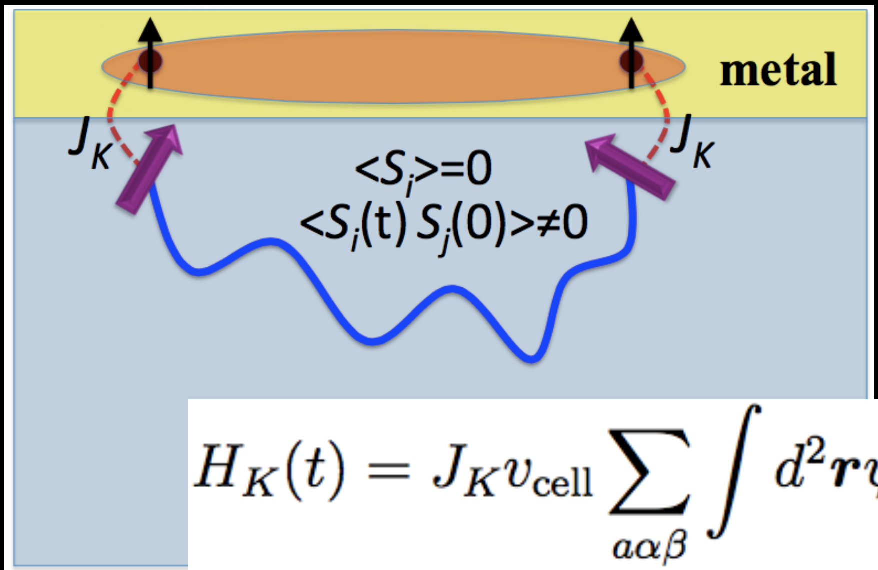

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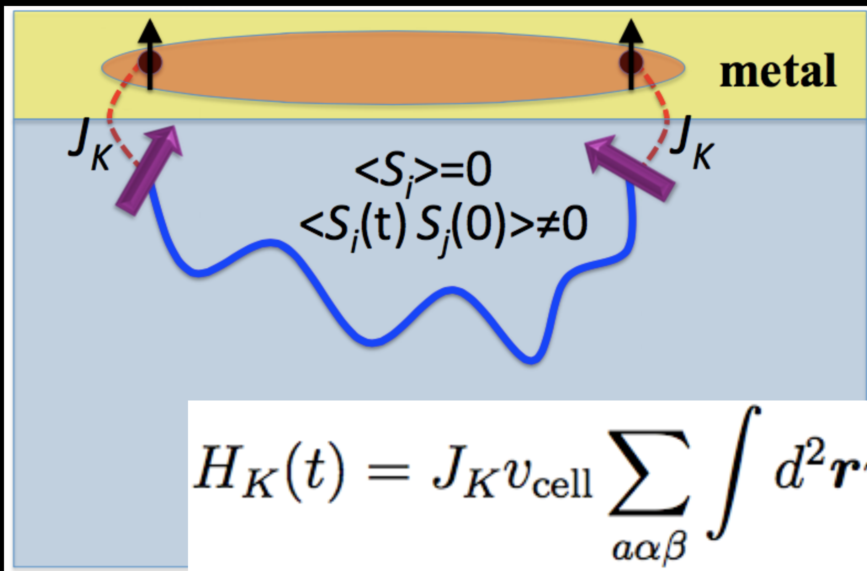

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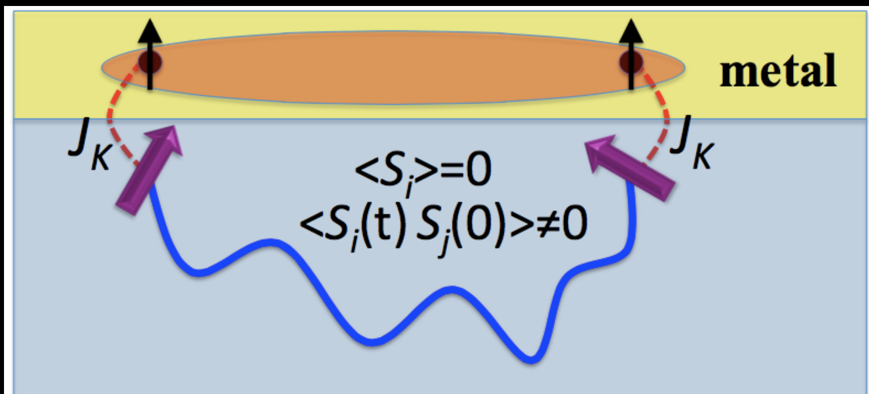

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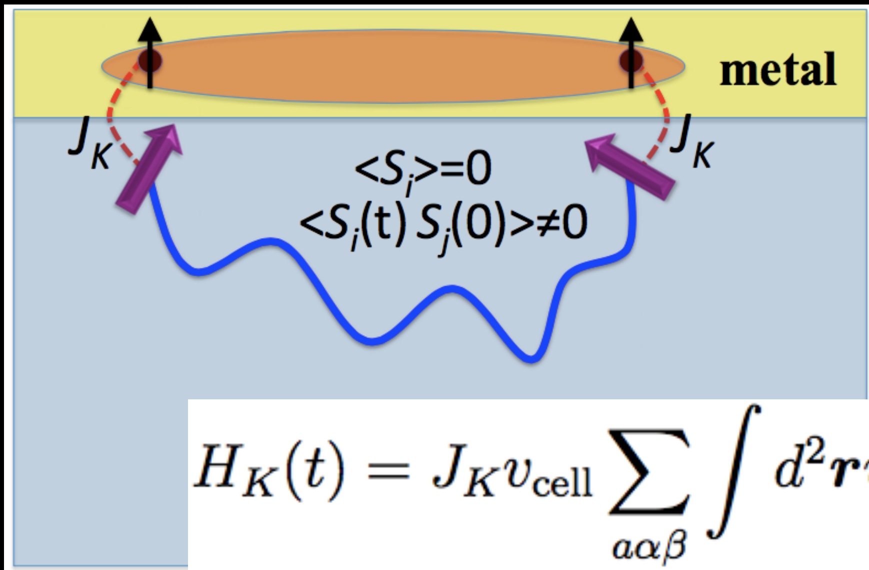

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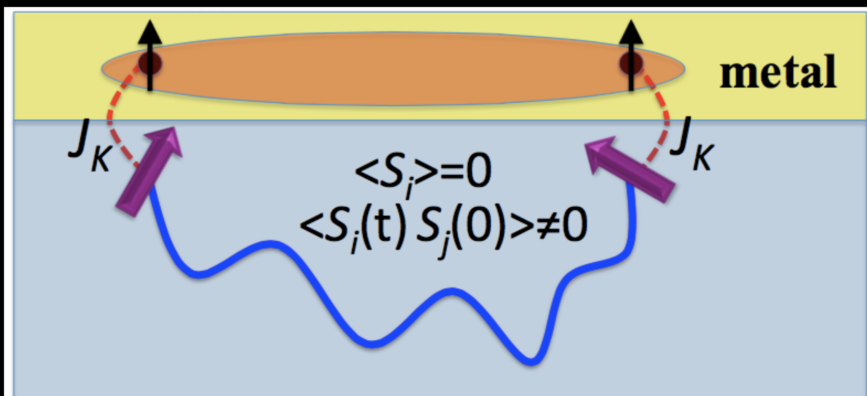

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Electrons are not magnetic monopoles

- Attractive equal-spin interaction!

$$-J_K^2 D(q) \left(\vec{\sigma}_{\alpha\beta} \times \hat{\boldsymbol{q}}\right) \cdot \left(\vec{\sigma}_{\alpha'\beta'} \times \hat{\boldsymbol{q}}\right)$$

Dealing with interacting electrons?

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2/2\hbar) \sum_{ab} \int dt' \int d^2\boldsymbol{r} d^2\boldsymbol{r}' s_a(\boldsymbol{r},t) \langle S_a(\boldsymbol{r},0,t) S_b(\boldsymbol{r}',0,t') \rangle s_b(\boldsymbol{r}',t')$$

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$$T_c \sim \omega_s e^{-1/\lambda}$$

Selection Rule Dictated Odd-Parity

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- Pair binding problem with dipole-dipole interactio

$$V_{\rm dd} = \frac{1}{r^3}[\vec{S}_1 \cdot \vec{S}_2 - 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})] \propto \mathcal{R}^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2) \cdot \mathcal{S}^{(2)}(\boldsymbol{s}_1, \boldsymbol{s}_2)$$

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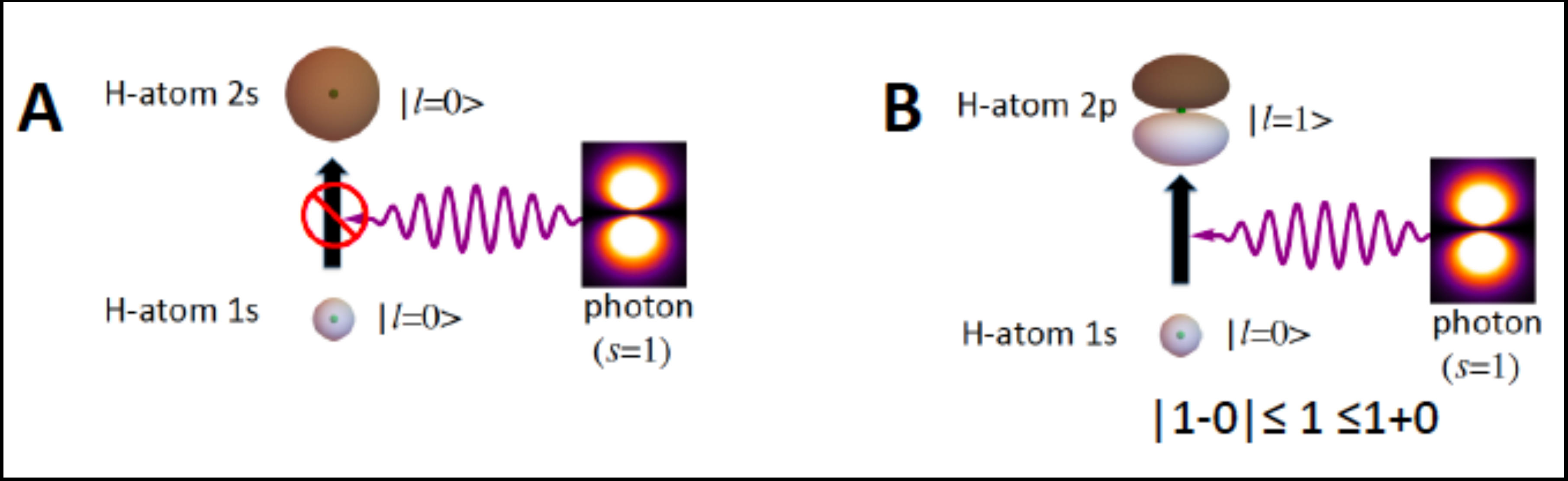

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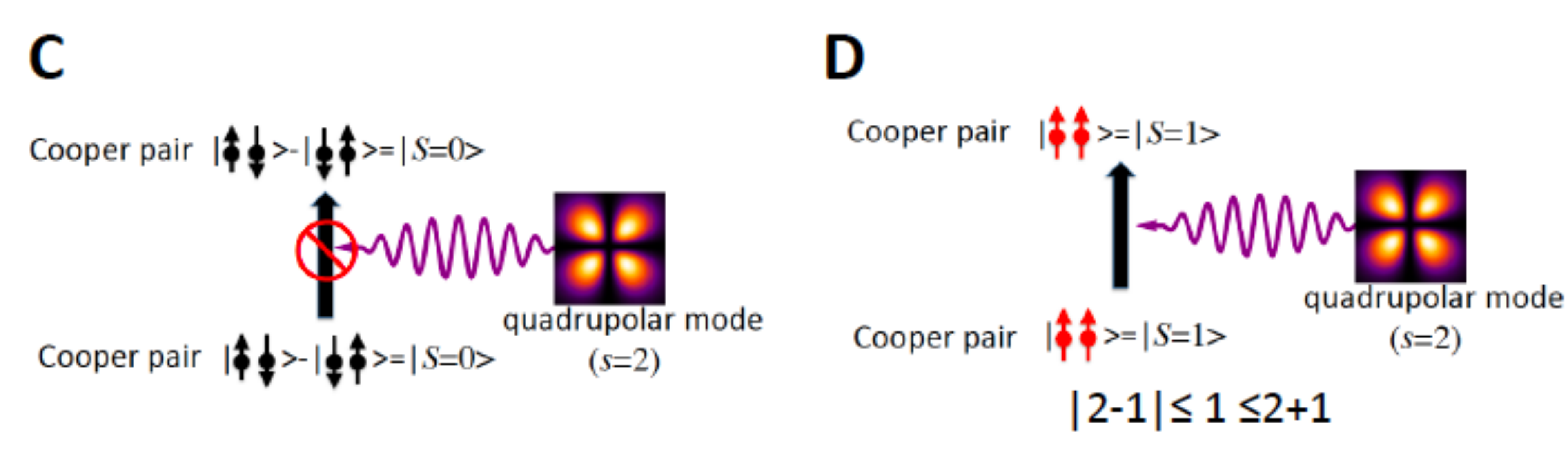

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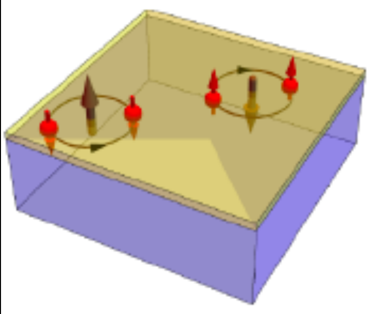

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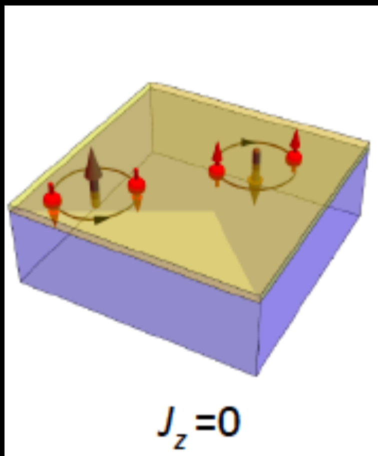

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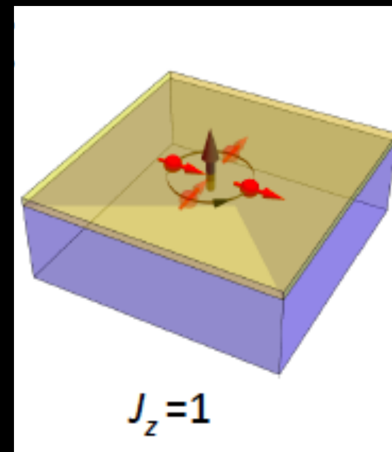

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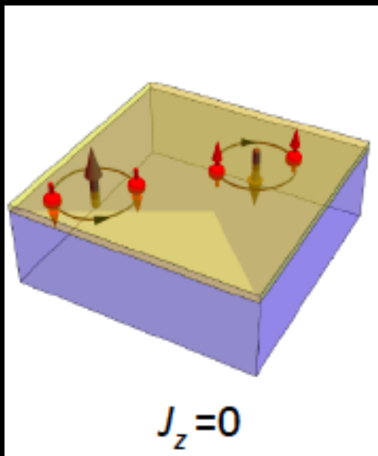

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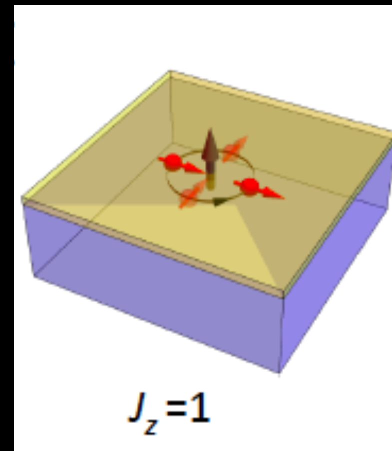

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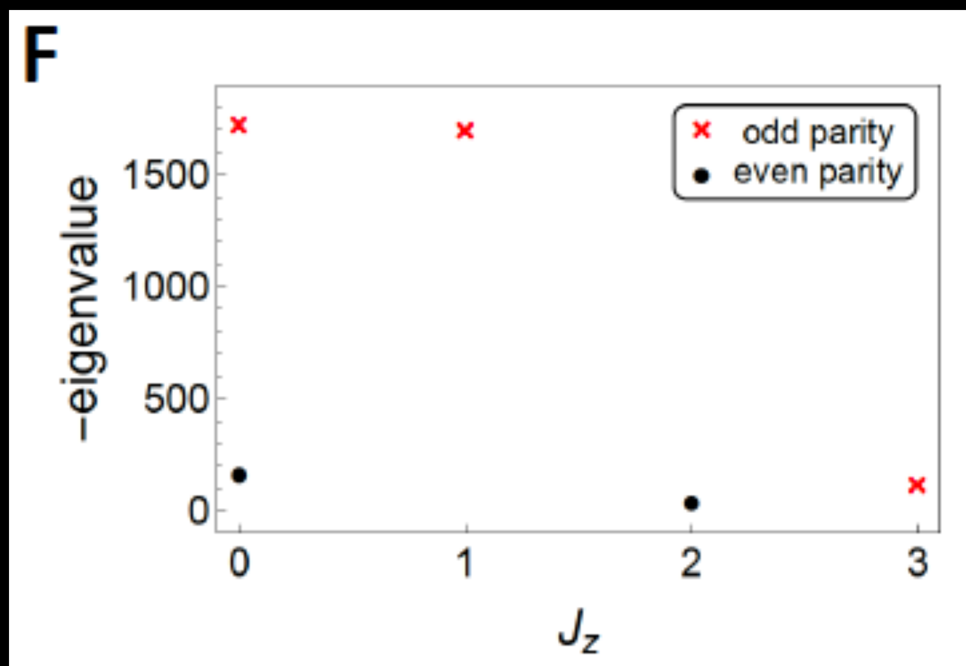

Can we persuade a material synthesis person?

Criteria for Metal

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- Structural
  - Lattice match
    - ➡ $A_2B_2O_7$
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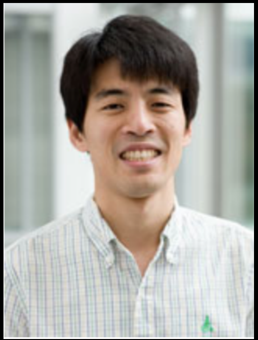
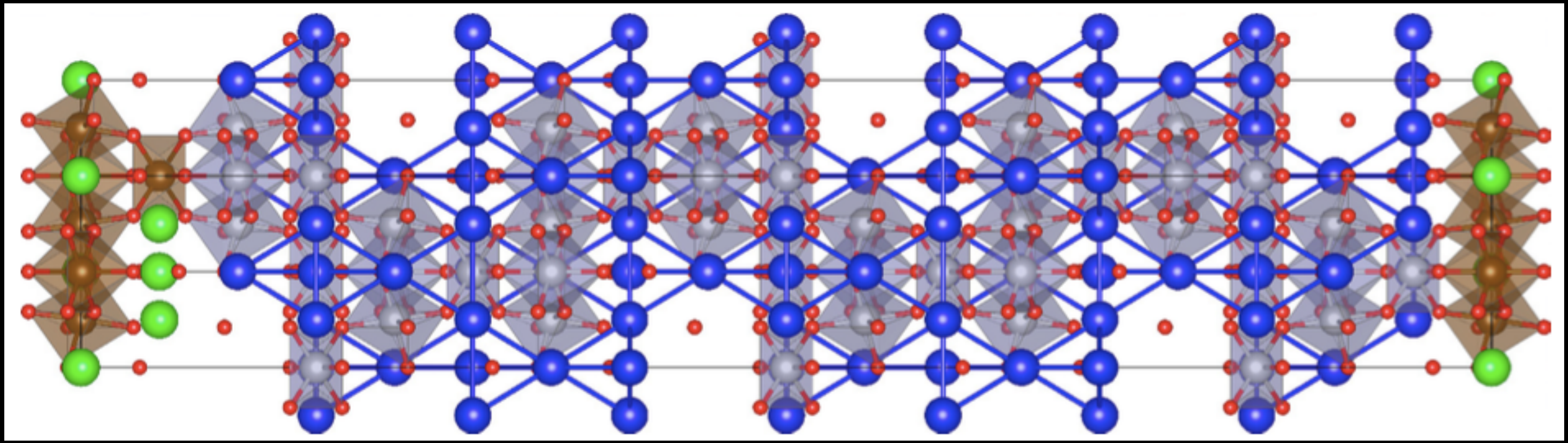
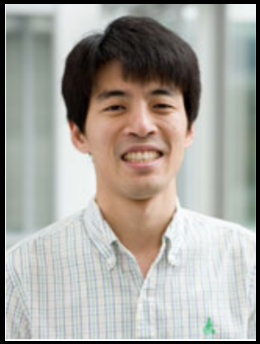
- Electronic
  - Simple isotropic Fermi surface
  - Wave function penetration
  - Odd-# FS around high symmetry points



$$Pr_2Zr_2O_7/Y_2Sn_{2-x}Sb_xO_7\ (111)$$





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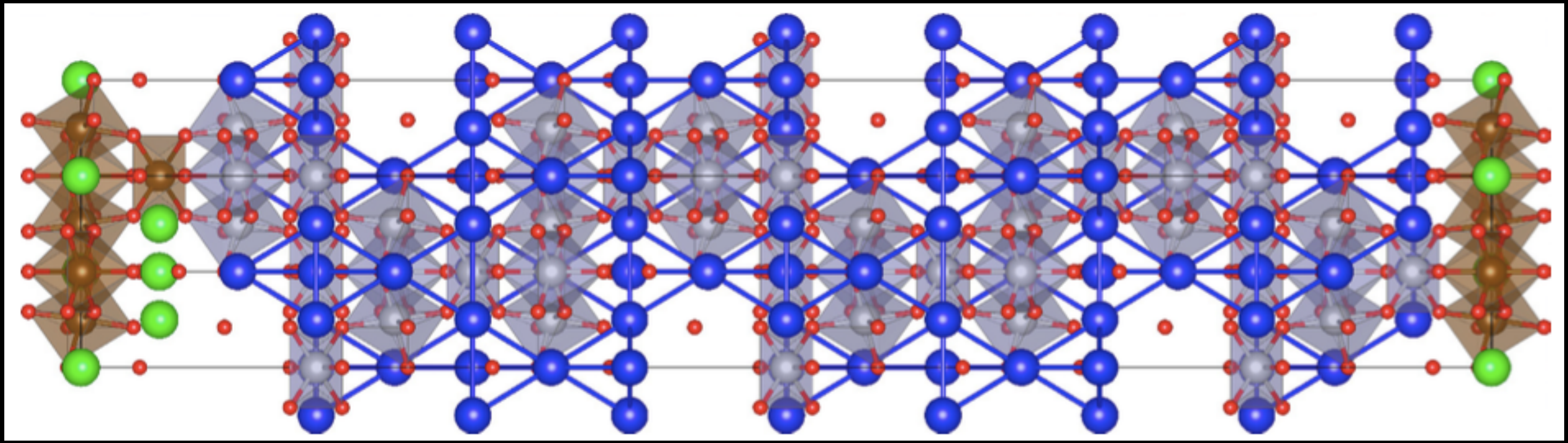

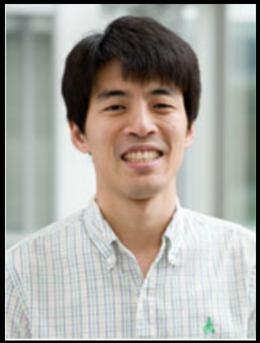

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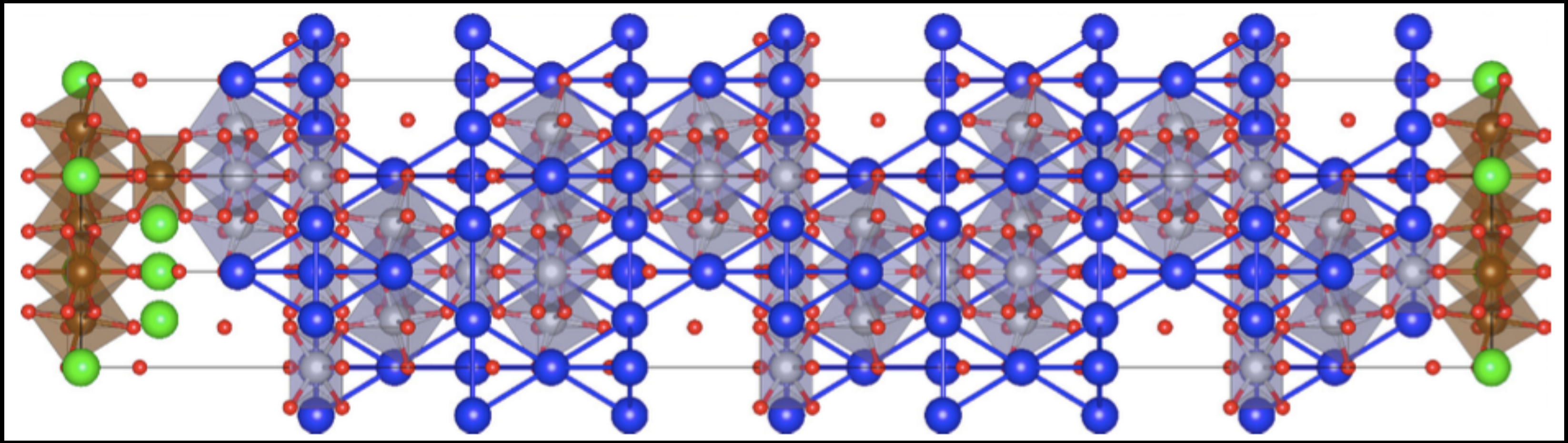
Non-magnetic

s-electrons:
large overlap,
isotropic FS.



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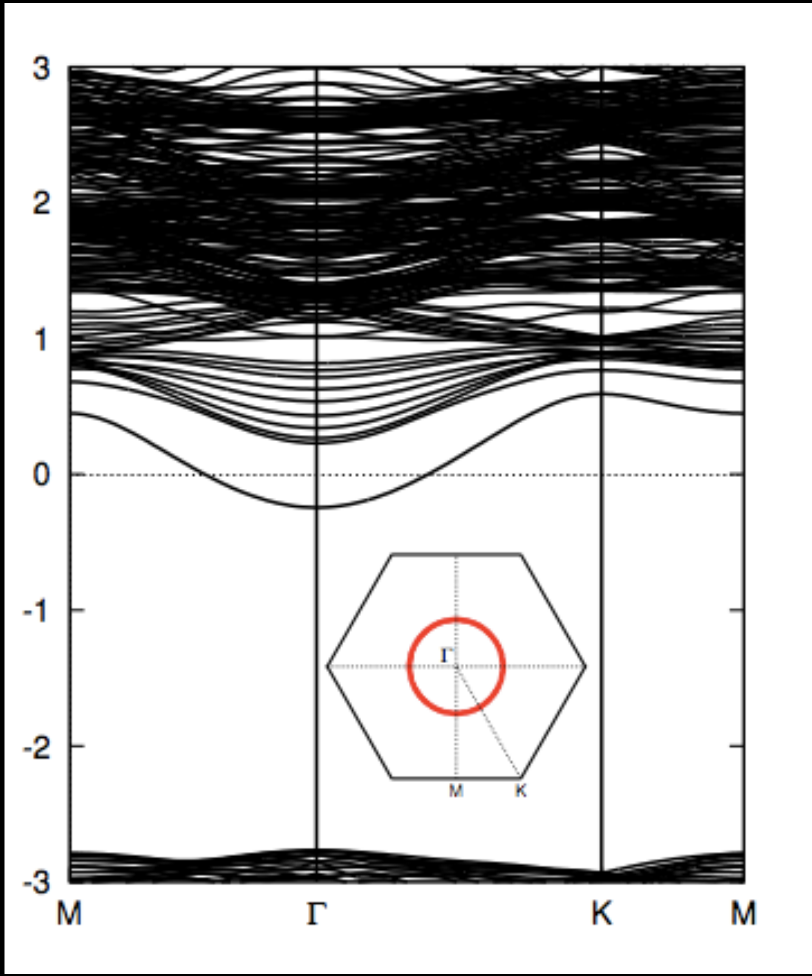

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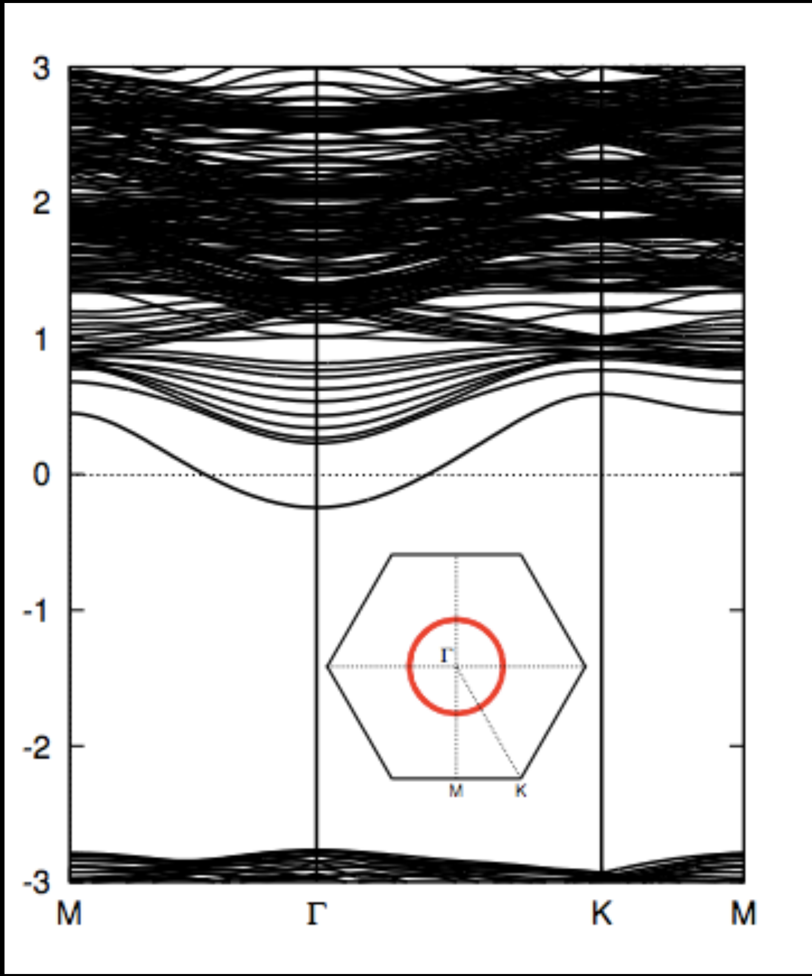
- Isotropic single pocket centered at Γ-point



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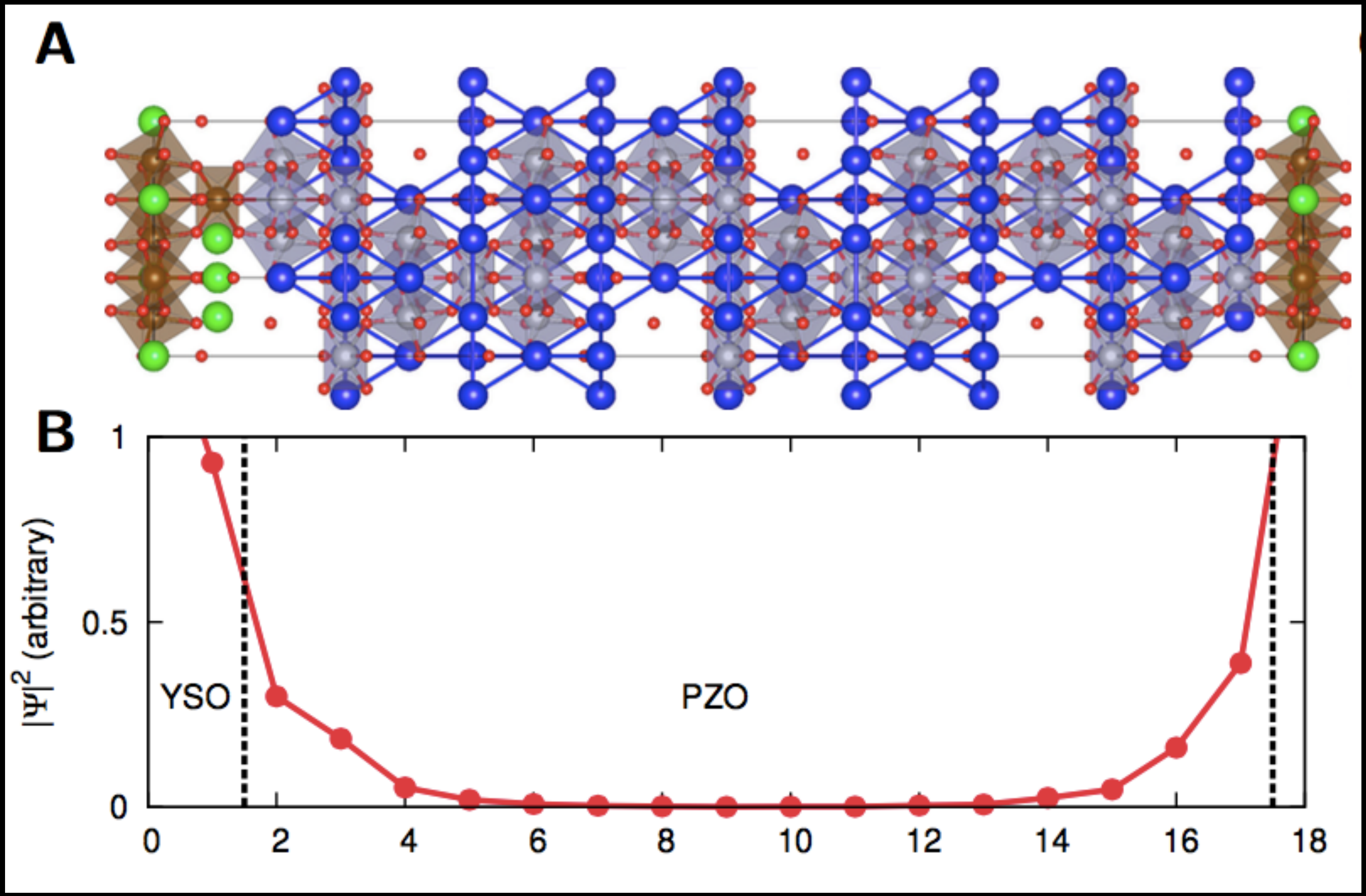

Earlier Proposal: Excitonic mechanism

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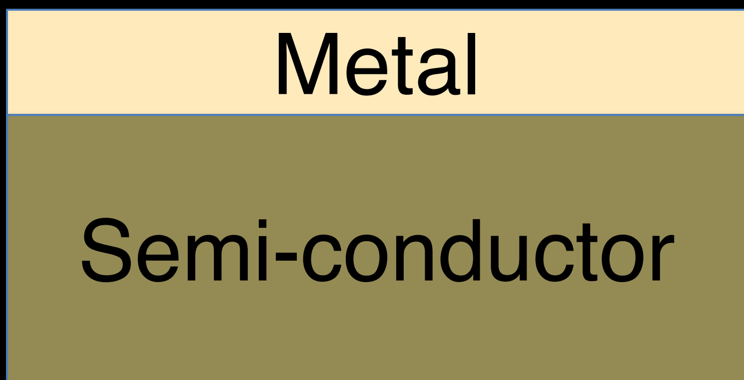

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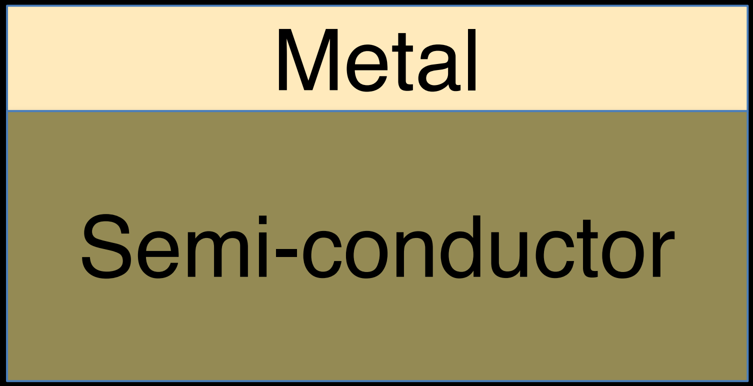
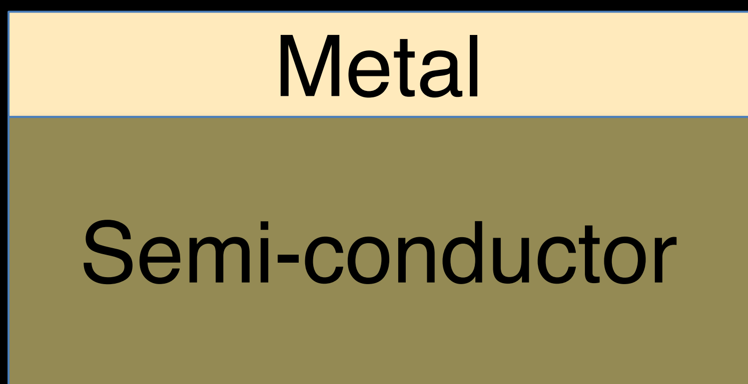


- Unstable against exchange.

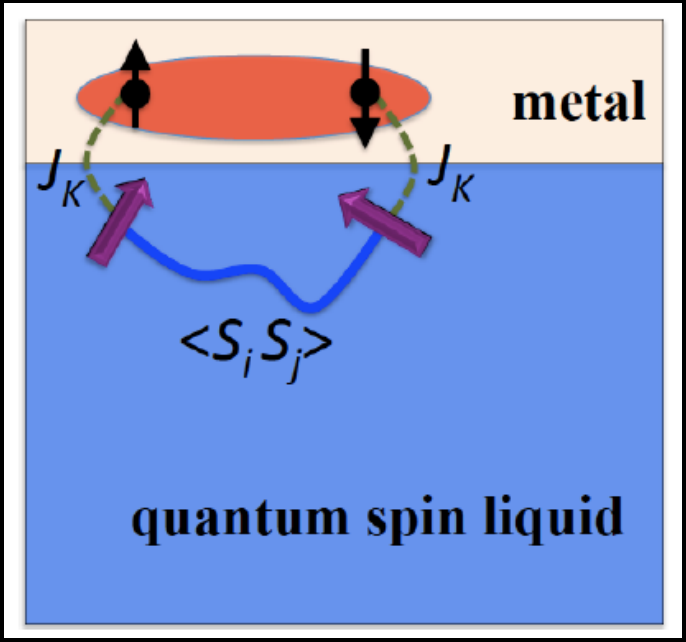
Earlier Proposal: Excitonic mechanism



- Unstable against exchange.
- Intrinsically s-wave.

Little (64), Ginzburg (70), Bardeen (73)

Topological Superconductivity in Metal/Quantum-Spin-Ice Heterostructures



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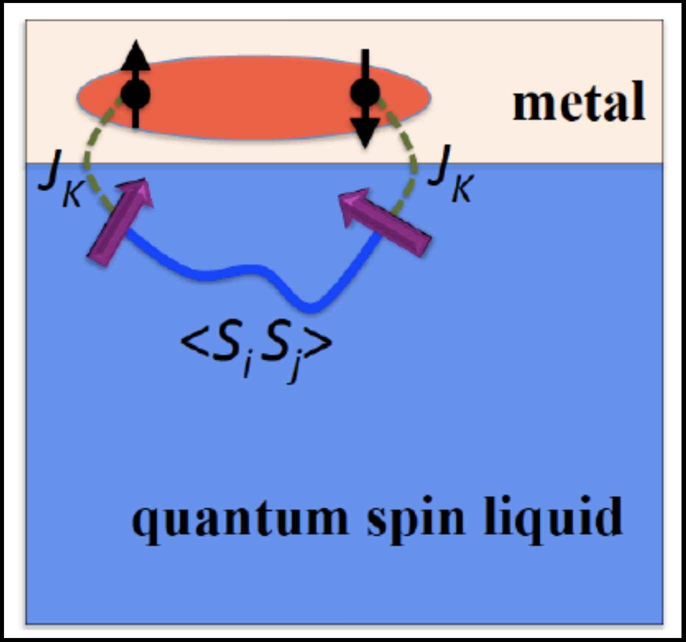

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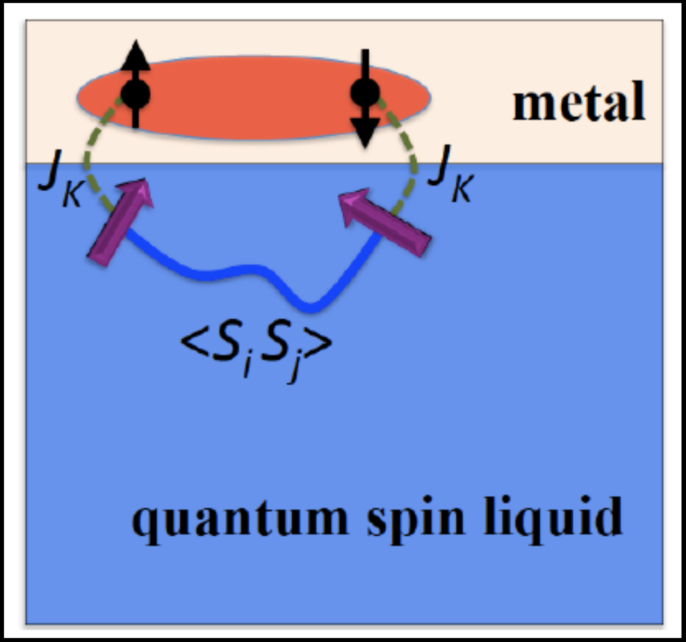

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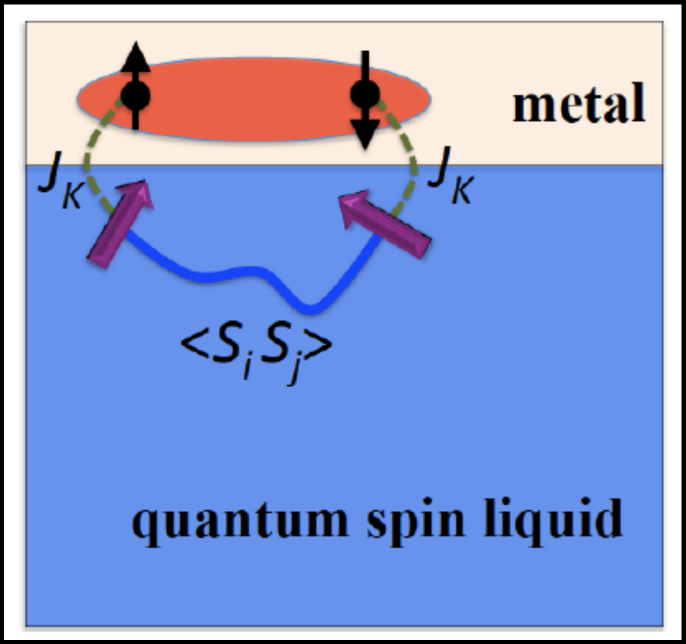
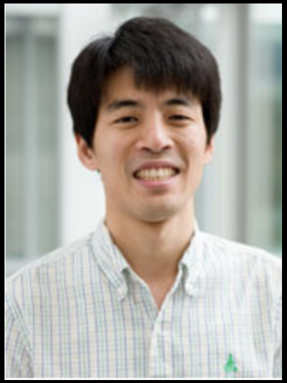


- Topological superconductor riding on QSL
- Selection Rule Dictated Intrinsic Topo SC.
- Substantial phase space.

Acknowledgements

Jian-huang She



Choonghyun Kim

Criag Fennie

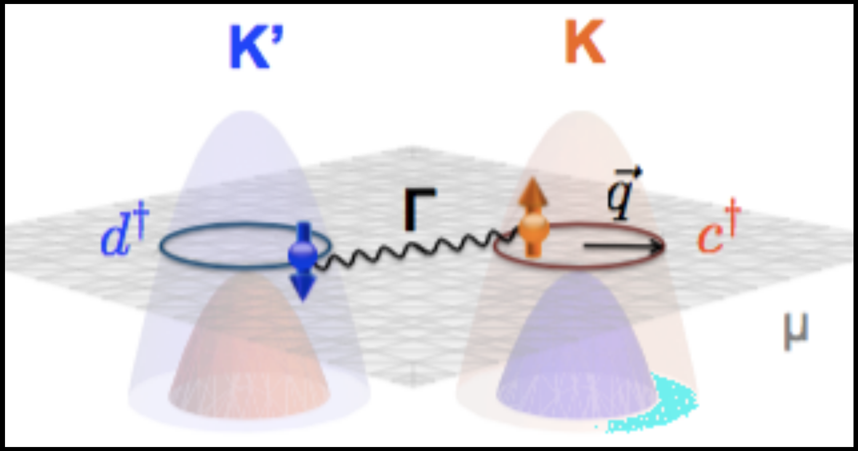
Michael Lawler

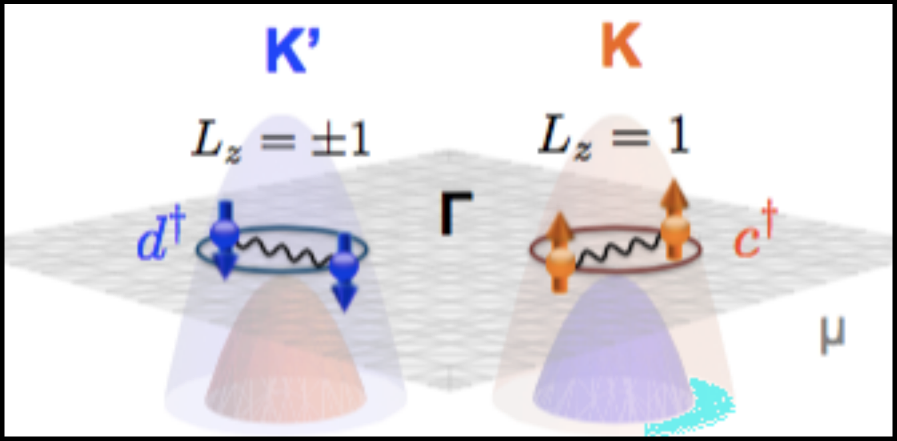
Funding: DOE, CCMR (NSF)

Strategy II

Manipulate the band structure

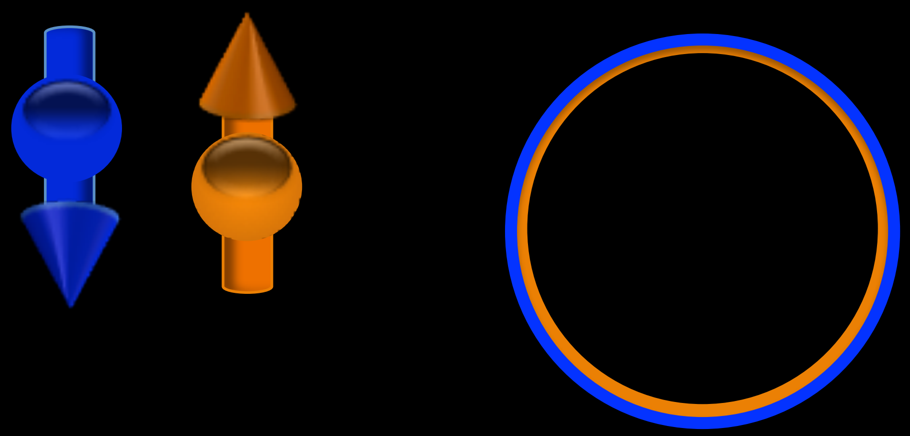
Topological superconductivity in group-VI TMDs





Yi-Ting Hsu, Abolhassan Vaezi, E-AK (in preparation)

Spin-degenerate Fermi surface



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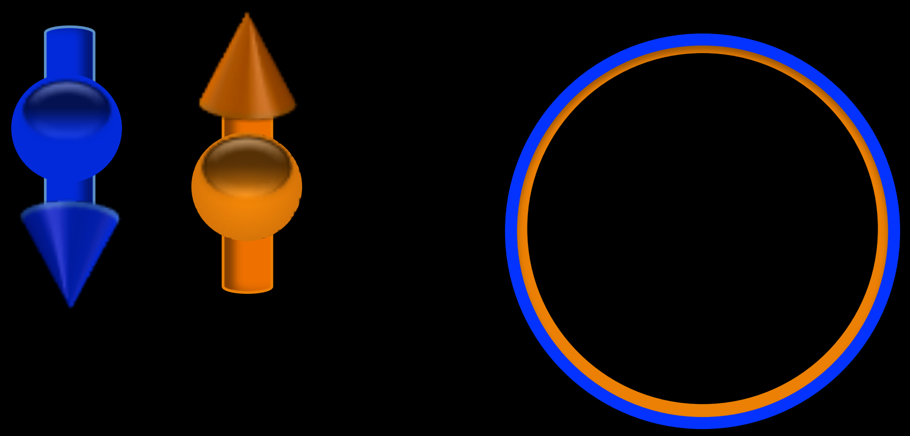

Singlet superconductor

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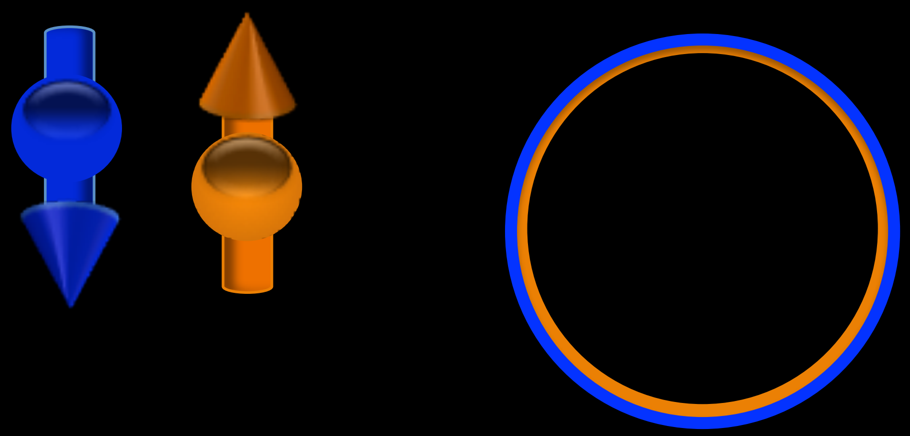
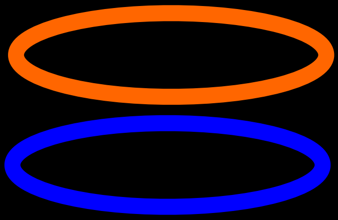


Singlet superconductor

Q. What if the band structure is spin-split?

Spinless fermion via real space splitting

Spinless fermion via real space splitting



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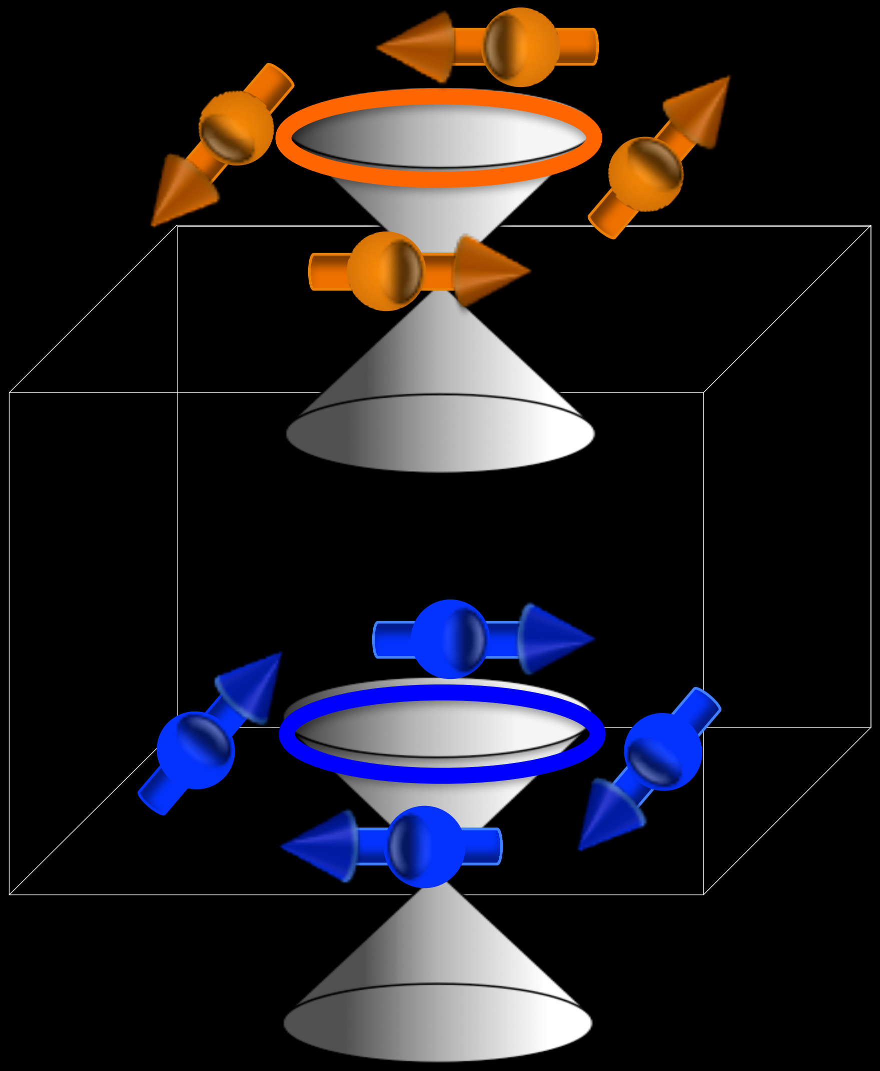
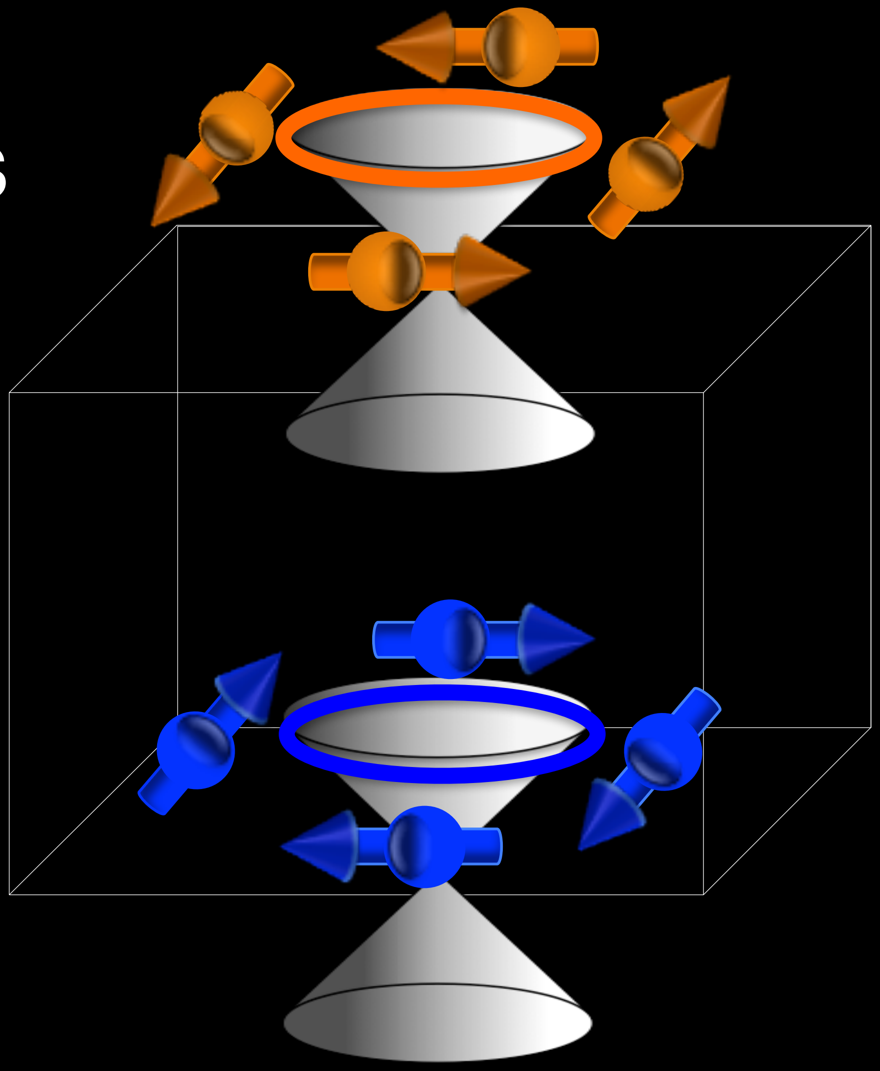


Spinless fermion via real space splitting

- TI surface states



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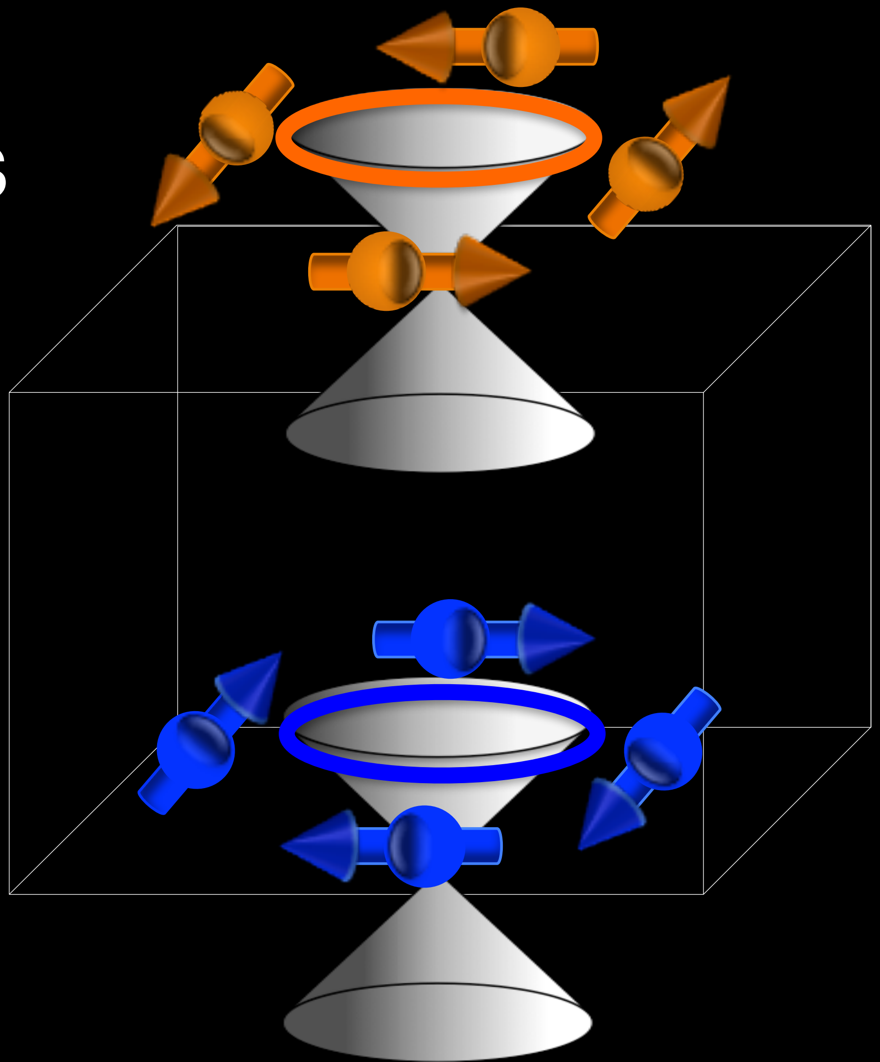

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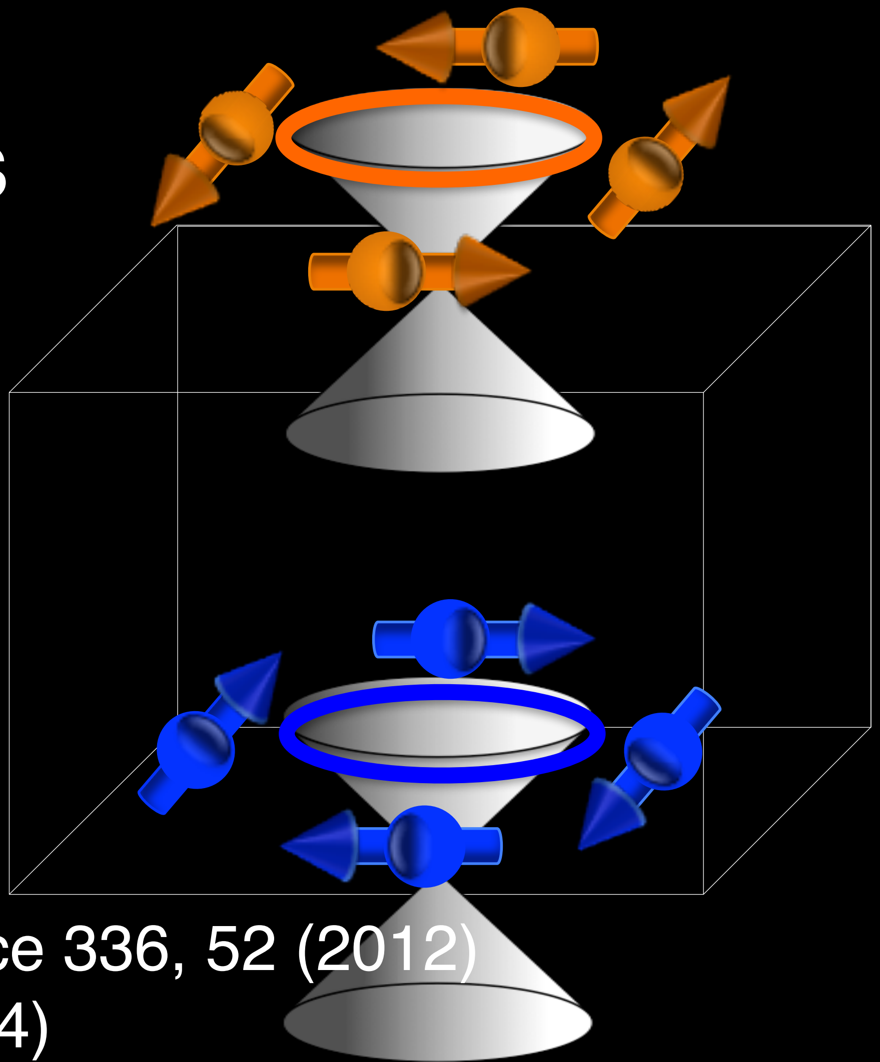
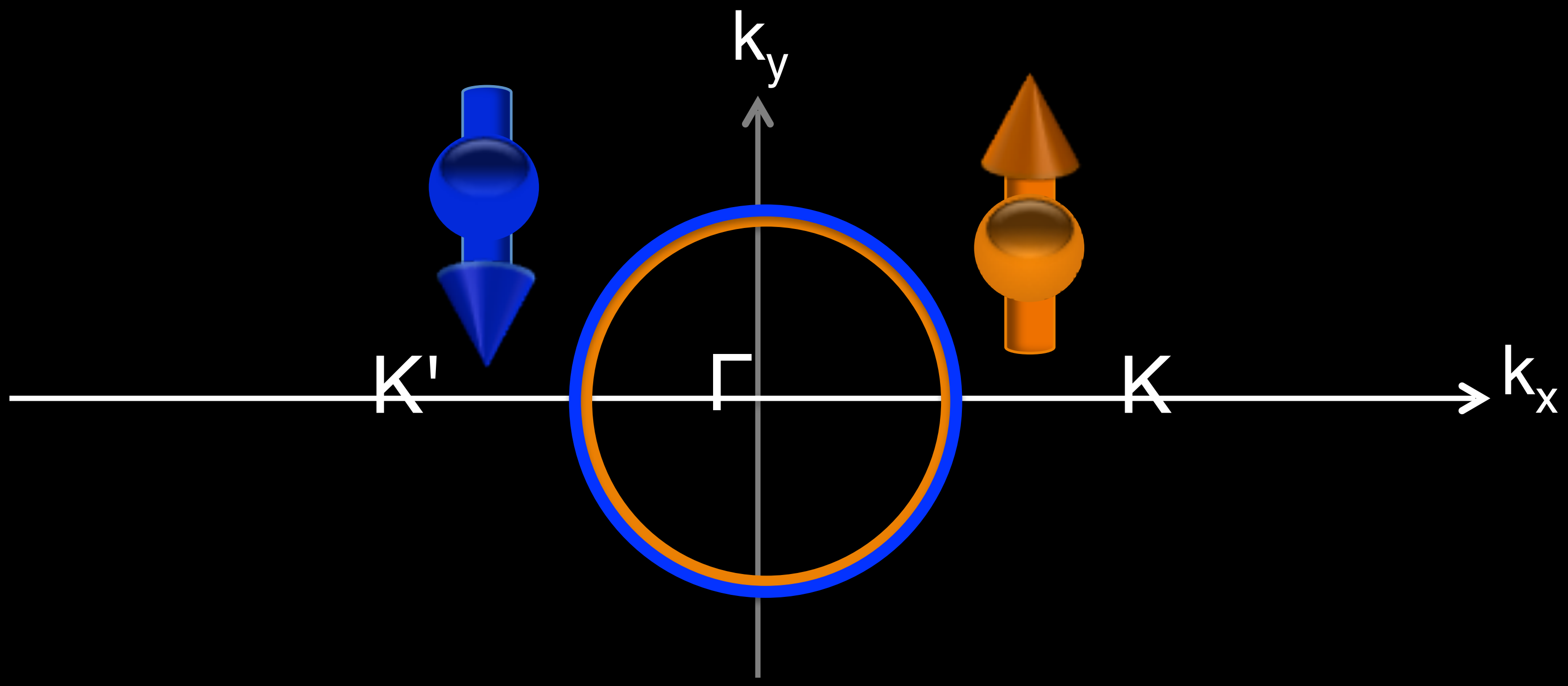
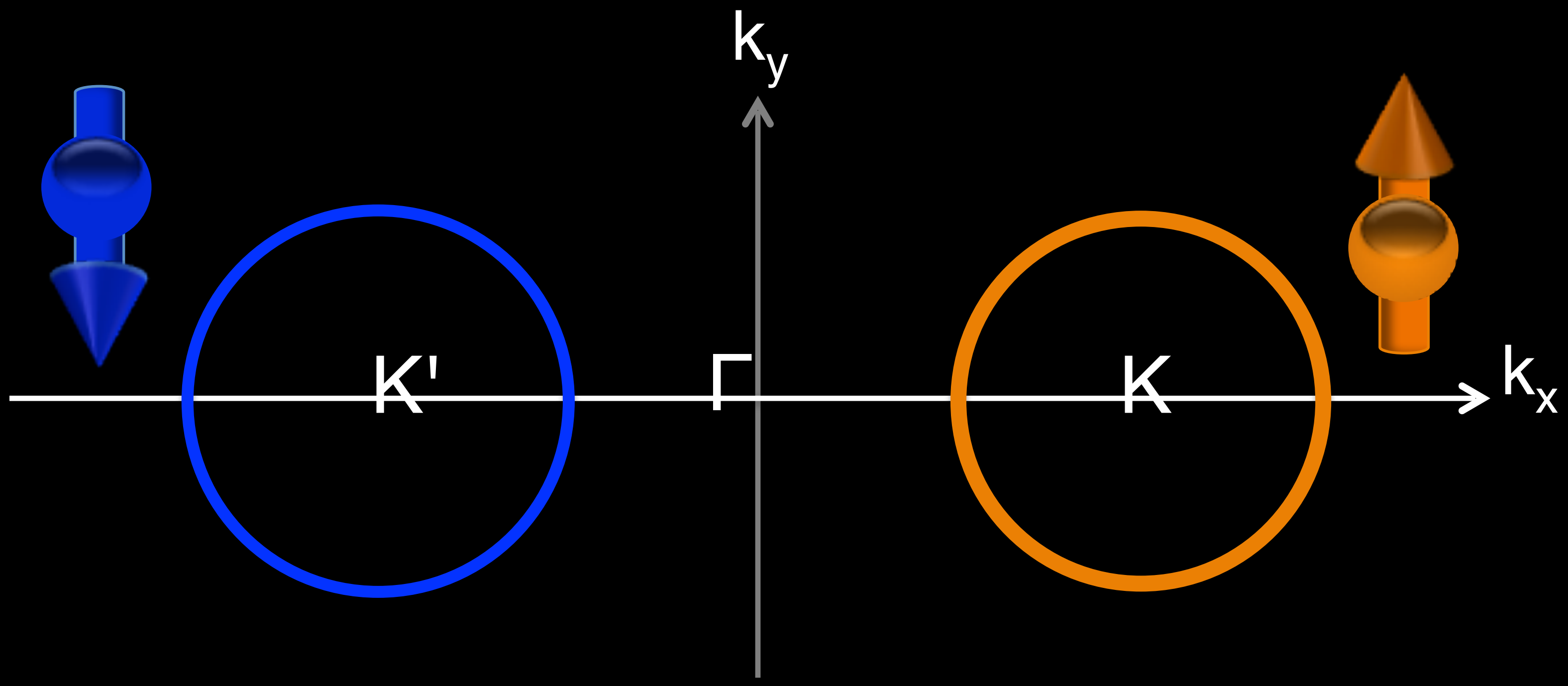


Fu & Kane, PRL (2008)
Experiments: Wang et al Science 336, 52 (2012)
Xu et al, Nat.Phys 10, 943 (2014)

Spinless fermion via k-space splitting?



Spinless fermion via k-space splitting?



Monolayer group VI TMD's

MoS_2, WS_2, $MoSe_2$, WSe_2

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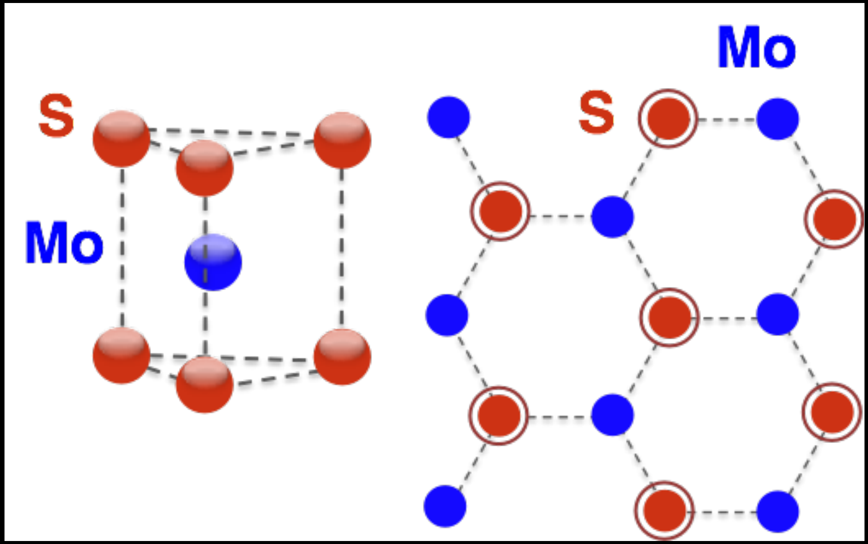

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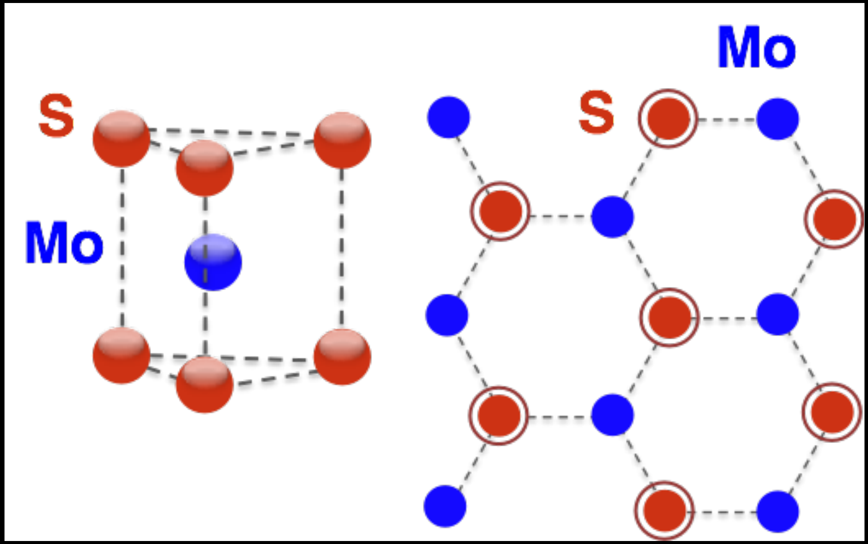

- **Non-centro symmetr**

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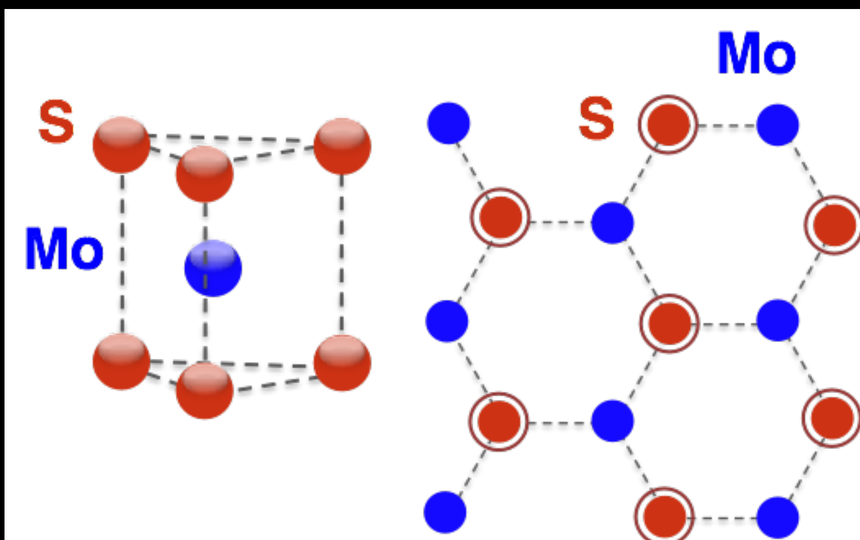

- **Non-centro symmetr**

➡ Direct Gap ~2eV

➡ Dresselhaus spin-orb

Band-selective spin-splitting

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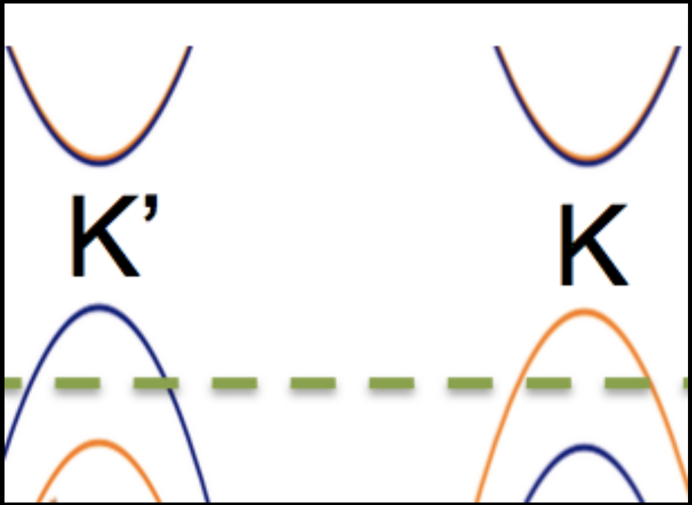

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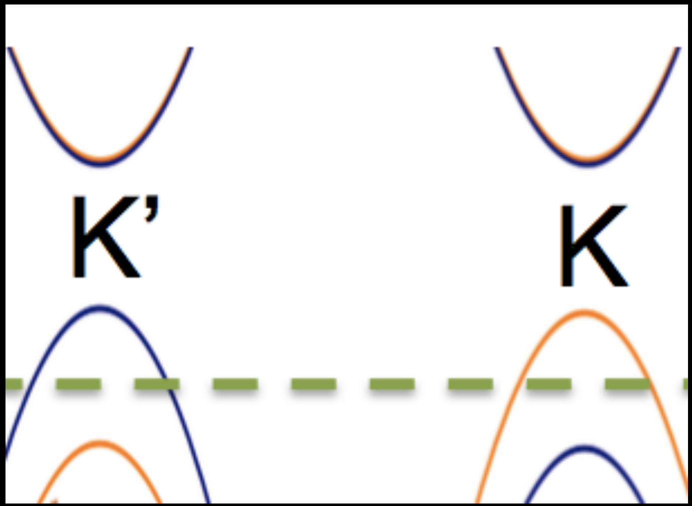
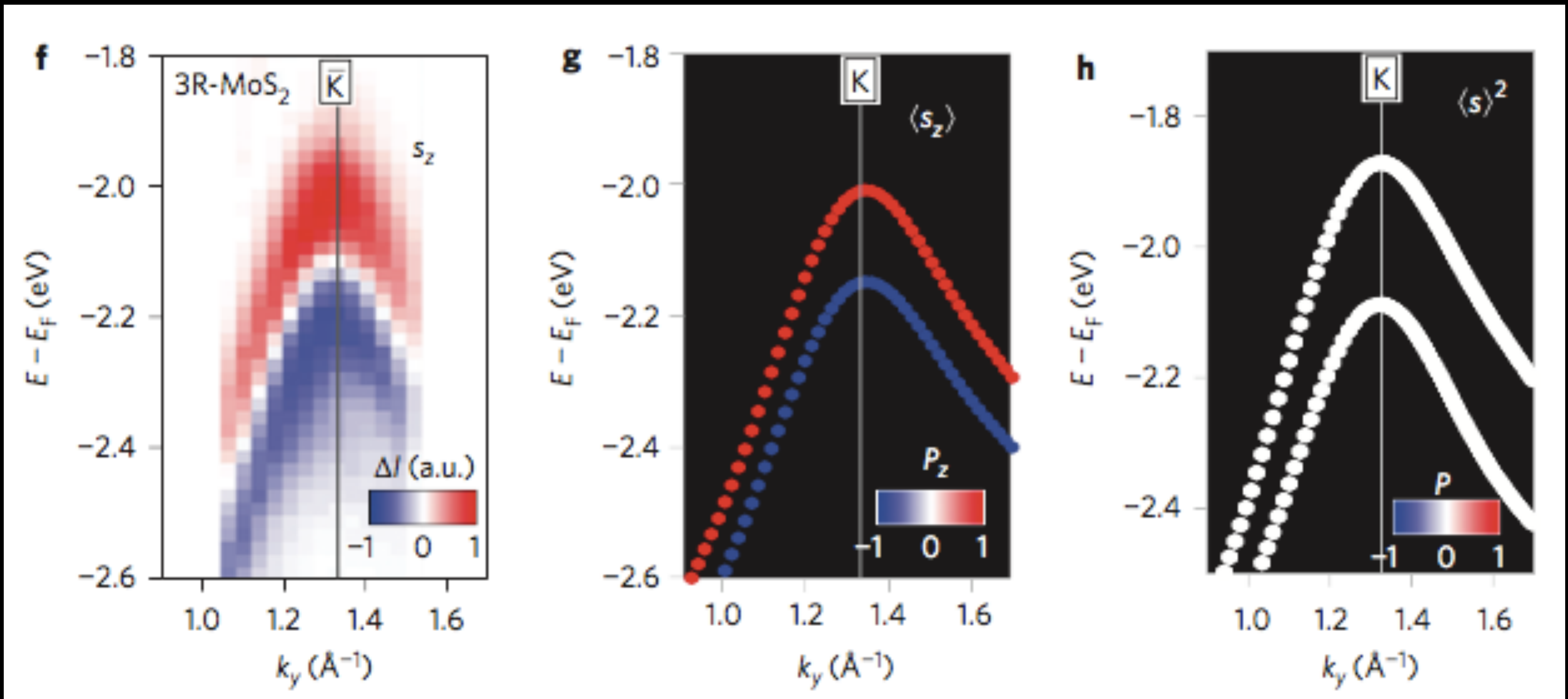
150~460meV





Iwasa group N. Nano (2014)

k-space spin-split FS?

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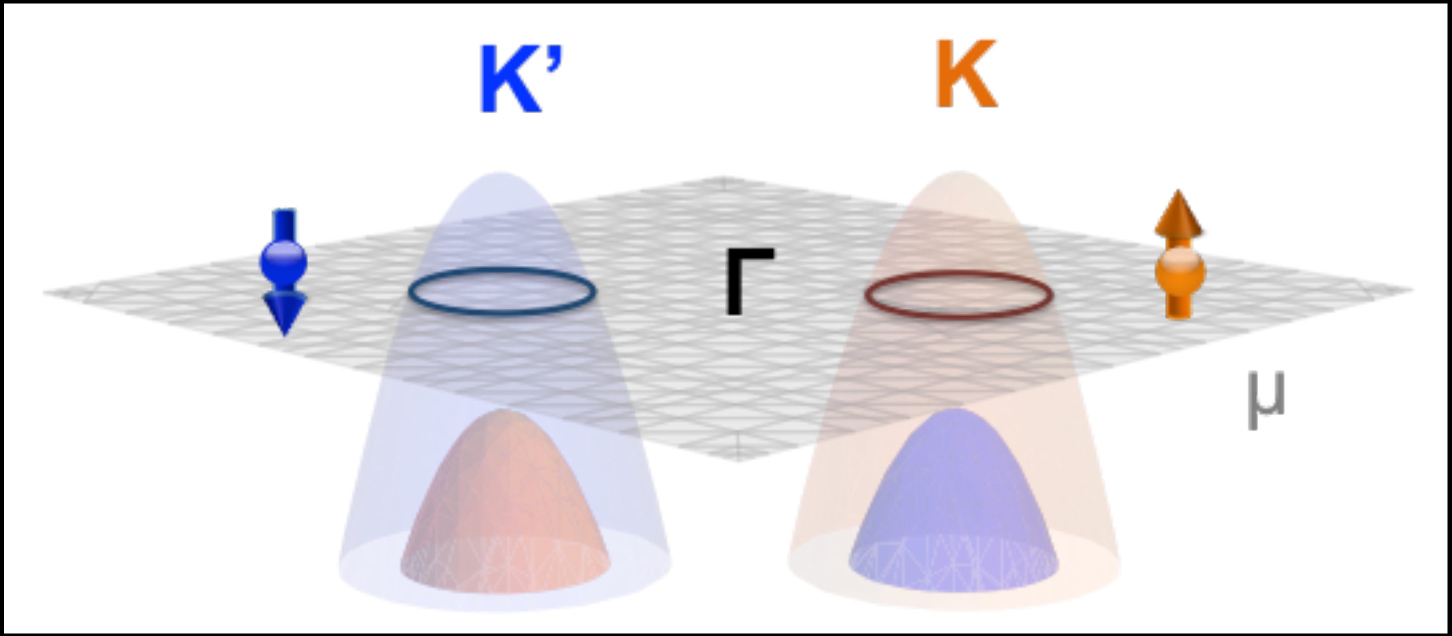
p-doped group VI- TMD!



Juice for superconductivity?

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- d electrons => expect correlation effects

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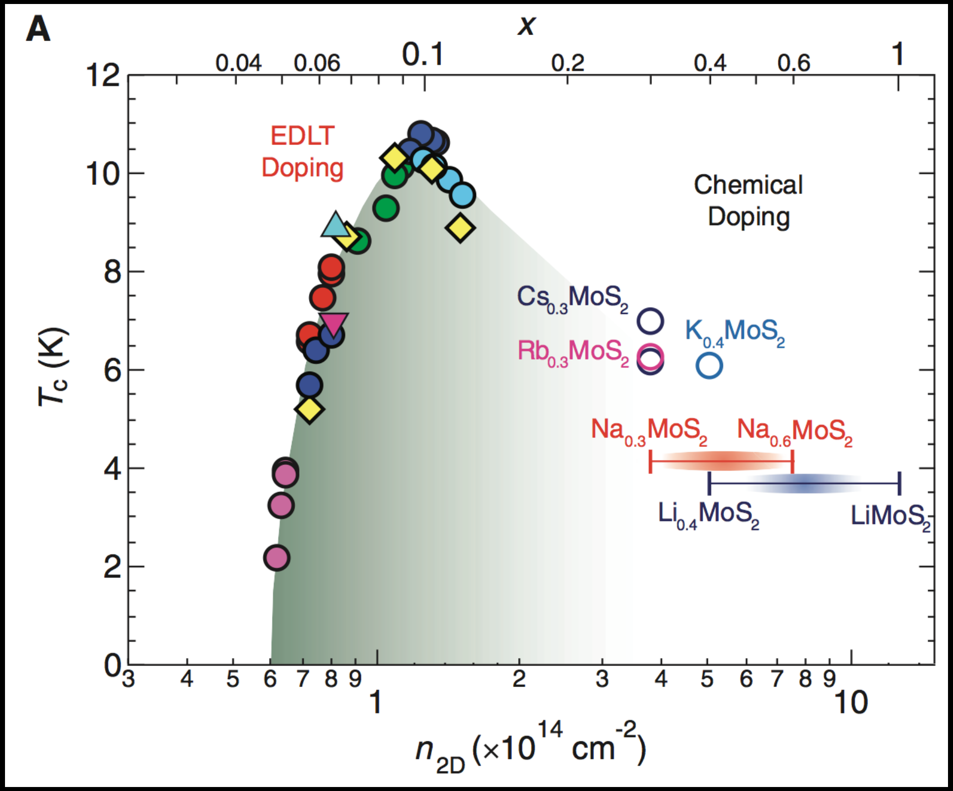
- n-doped

J.T.Ye *et al.* (Science 2012)



p-doped TMD

k-space spin-split Fermi surfaces

+

Moderate correlation (d-electron)

p-doped TMD

k-space spin-split Fermi surfaces

+

Moderate correlation (d-electron)

↓

Topological SC?

Yi-Ting Hsu

Mark Fischer

Abolhassan Vaezi

Model

- Kinetic t

$$H_0(\vec{q}) = at(\tau q_x \hat{\sigma_x} + q_y \hat{\sigma_y}) + \frac{\Delta}{2}\hat{\sigma_z} - \lambda\tau\hat{s_z} \otimes \frac{\hat{\sigma_z} - 1}{2}$$

- Repulsive interaction term

$$H'(W) = \sum_i U n_{i,\uparrow} n_{i,\downarrow}$$

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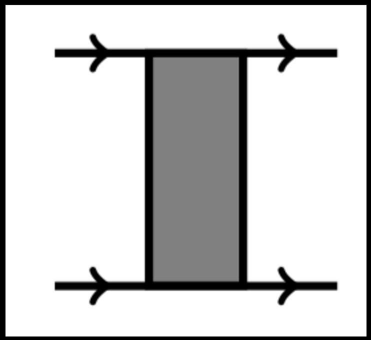
- Kohn-Luttiger: singularity in scattering amplitude $\Gamma(\vec{q})$  (Kohn &Luttinger 1965

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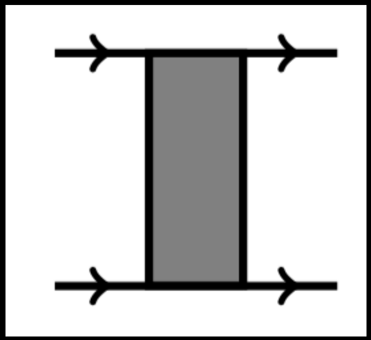

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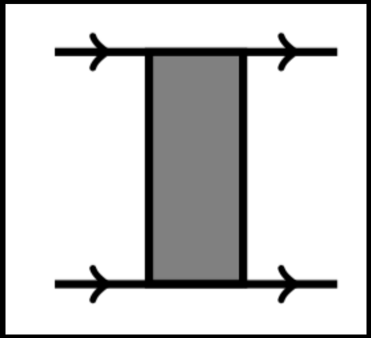
- Kohn-Luttiger: singularity in scattering amplitude $\Gamma(\vec{q})$  (Kohn &Luttinger 1965

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  : Fe-based SC, doped graphene, SrRu

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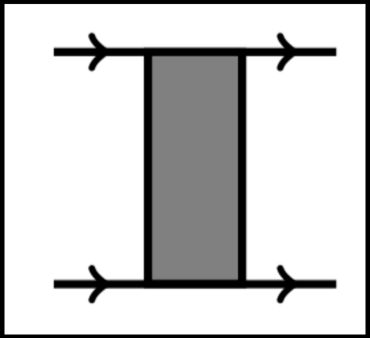

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➡Non-s wave

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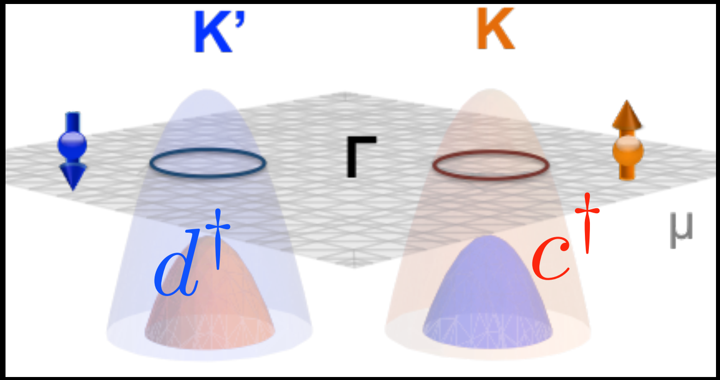
Chubukov & Nandkishore, Raghu & Kivelson (2008 - 2012)

Two-step RG on p-doped TMD

Step I: W -> Λ_0



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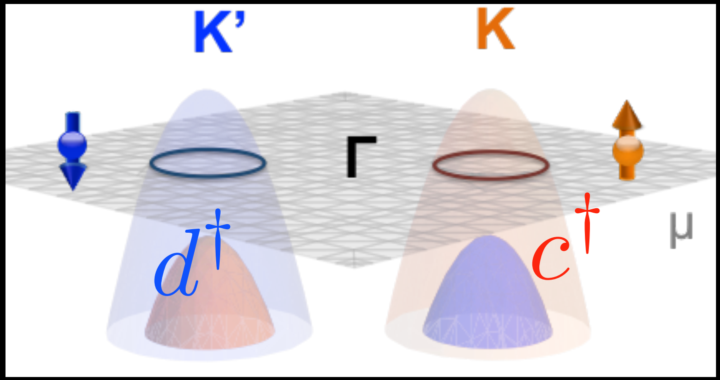
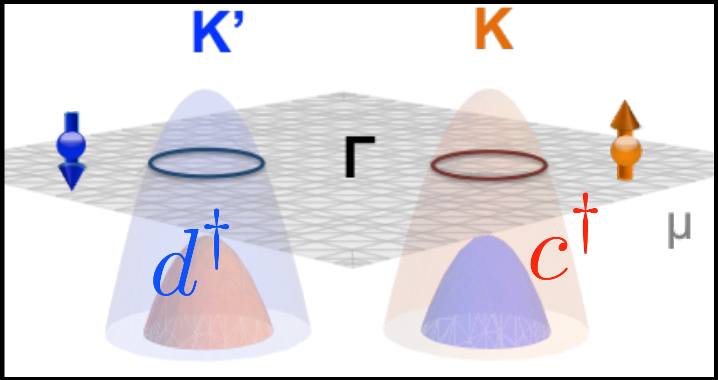


- At scale W: Microscopic model

Step I: W -> Λ_0



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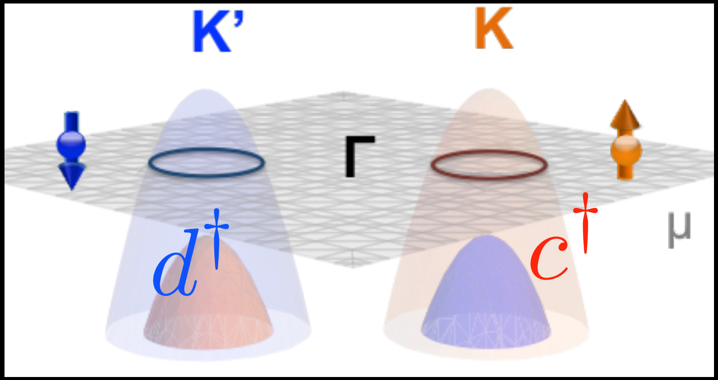
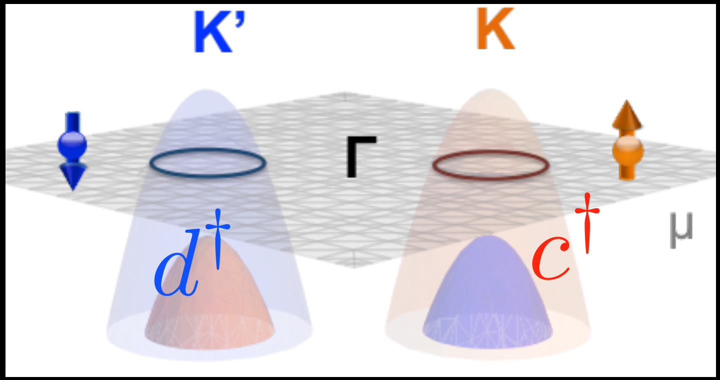


- At scale W: Microscopic model
- At scale Λ_0: Effective model

$$H'_{eff}(\Lambda_0) = \sum_{\vec{q},\vec{q}'}{}' g^{(0)}_{\text{inter}}(\vec{q},\vec{q}')c^\dagger_{\vec{q}'}d^\dagger_{-\vec{q}'}d_{-\vec{q}}c_{\vec{q}} + g^{(0)}_{\text{intra}}(\vec{q},\vec{q}')d^\dagger_{\vec{q}'}d^\dagger_{-\vec{q}'}d_{-\vec{q}}d_{\vec{q}} + (c \leftrightarrow d)$$

Step I: W -> Λ_0



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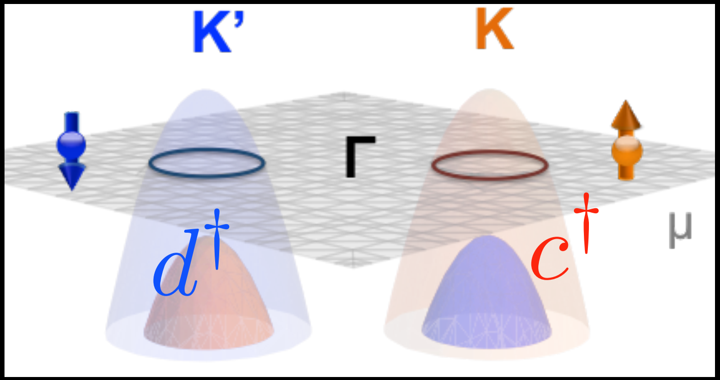

- $g_{intra,0}$ and $g_{inter,0}$ at two-loop

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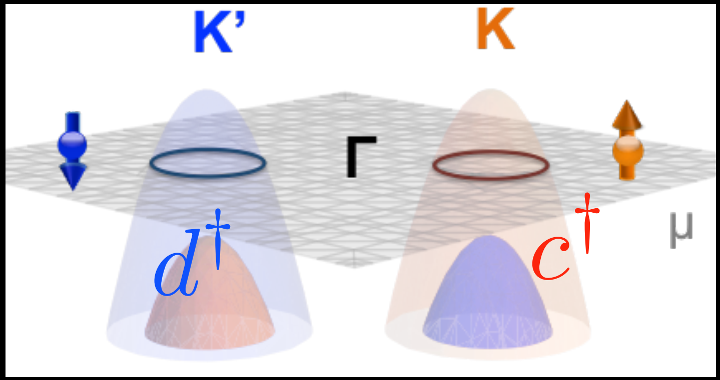

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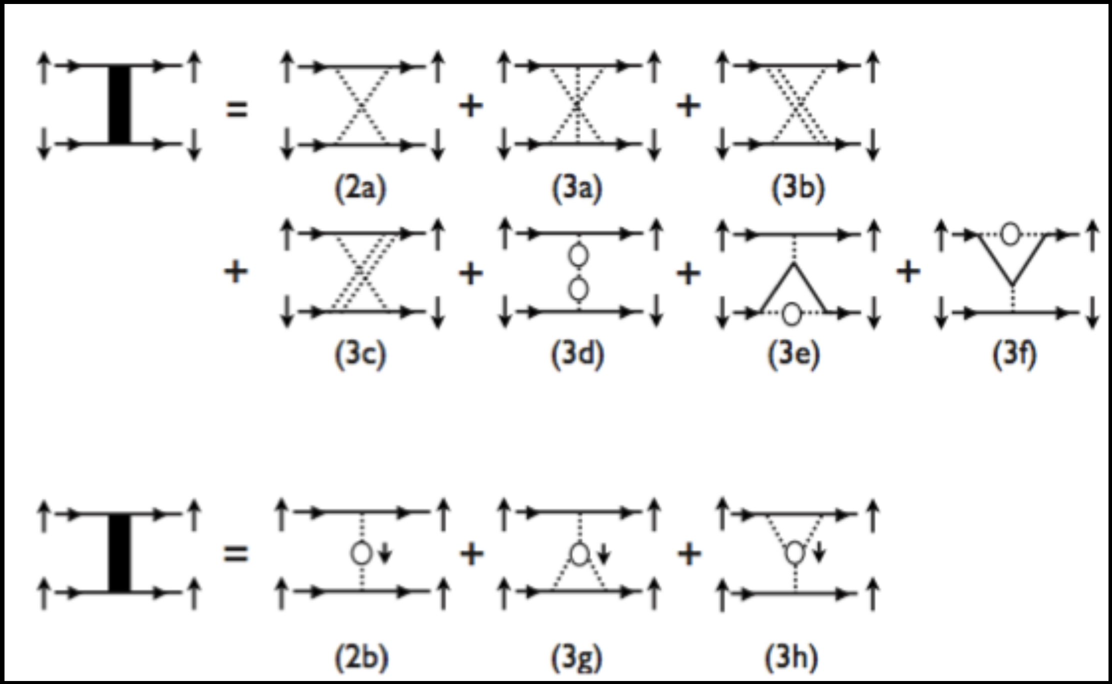

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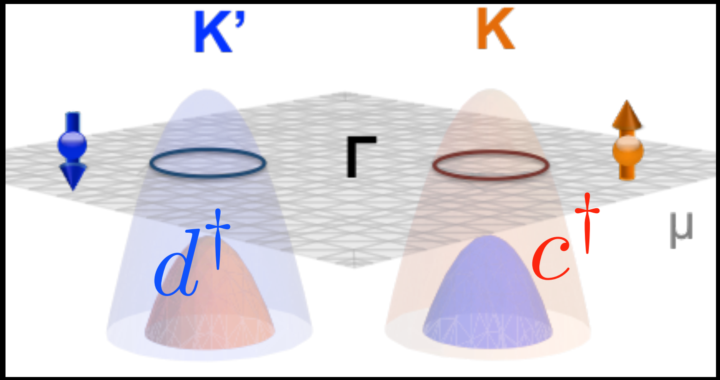

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$$g^{(0)}_{inter}(\vec{q},\vec{q}') = U + U^3 f_{inter}(\vec{q},\vec{q}')$$

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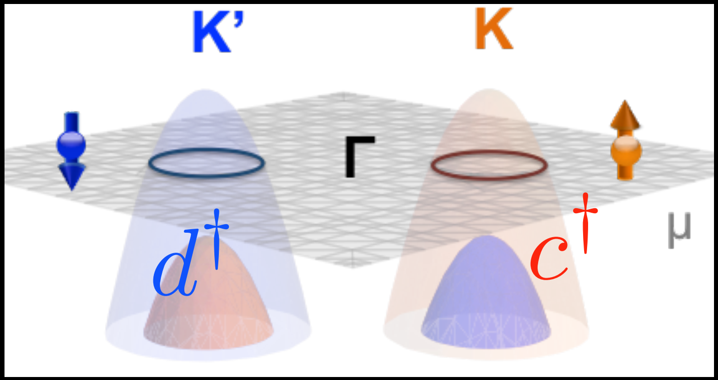

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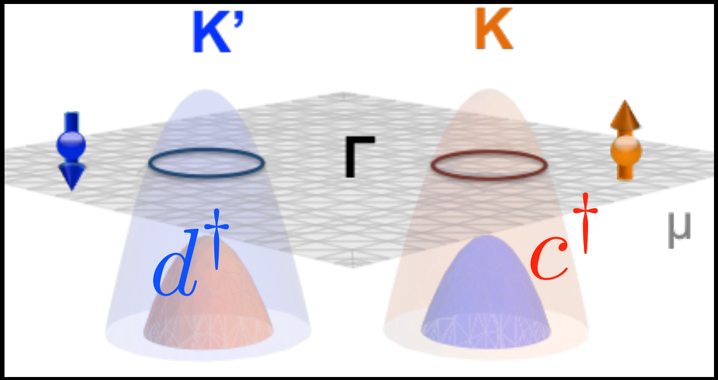

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- f's <0 -> $g^{(0)}$'s<0 in anisotropic channel

Step 2: Λ_0 ->0

- RG flow

- Divergence if $\lambda^{(0)} < 0$

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- RG flow $$\frac{d\lambda}{dy} = -\lambda^2$$

$$y \equiv \nu_0 \mathrm{Log}(\Lambda_0/\mathrm{E})$$

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Step 2: Λ_0 ->0

- RG flow $$\frac{d\lambda}{dy} = -\lambda^2$$ ⇒ $$\lambda(y) = \frac{\lambda^{(0)}}{1+\lambda^{(0)}y}$$

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Two possibilities

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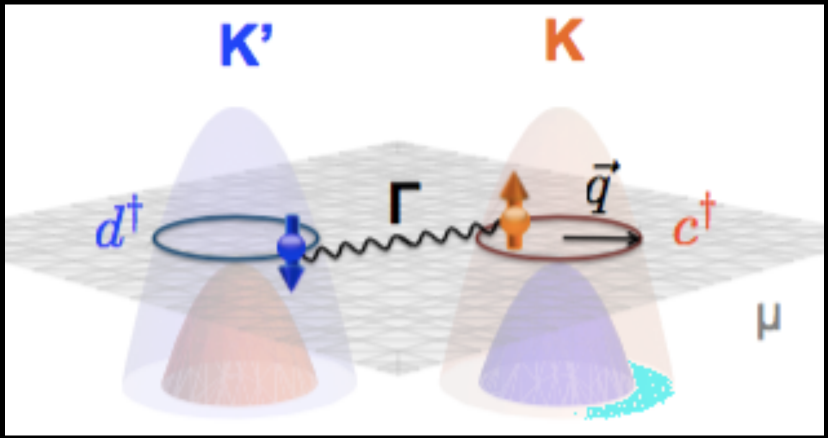
- Intra-pocket p+ip



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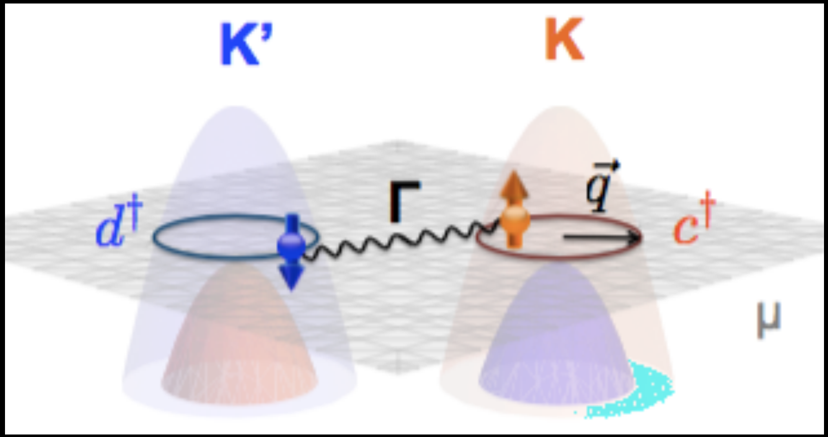
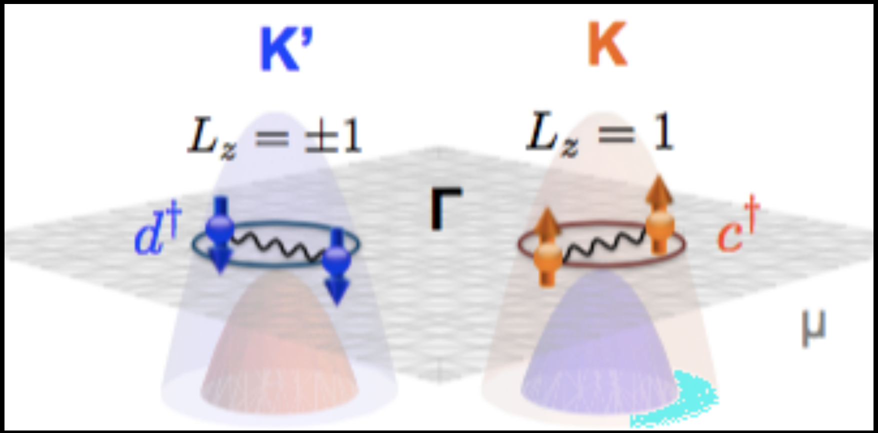


- Inter-pocket p'wave



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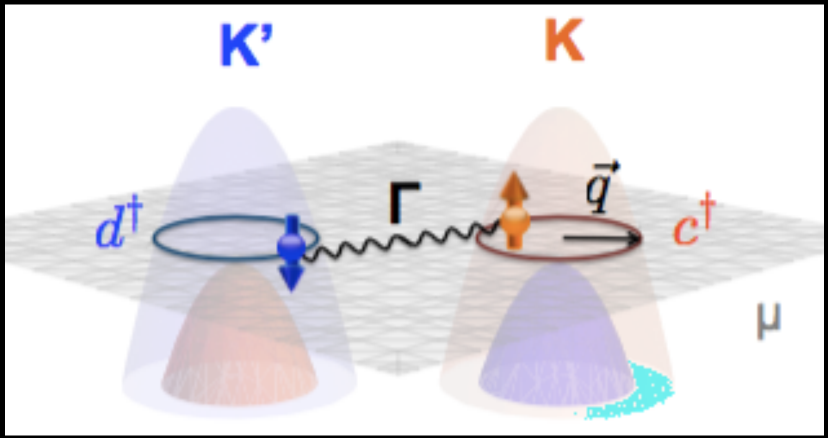

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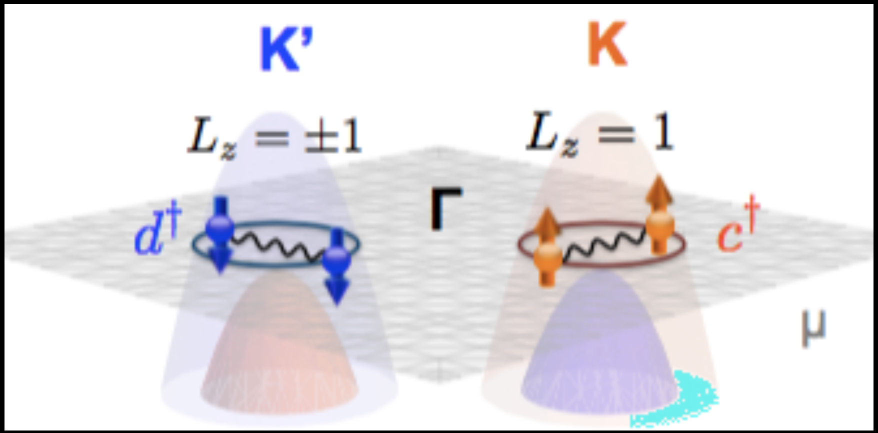

-T-breaking

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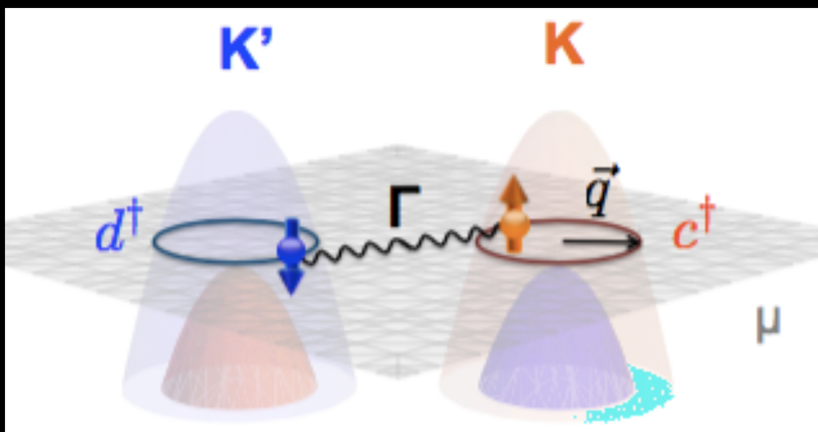

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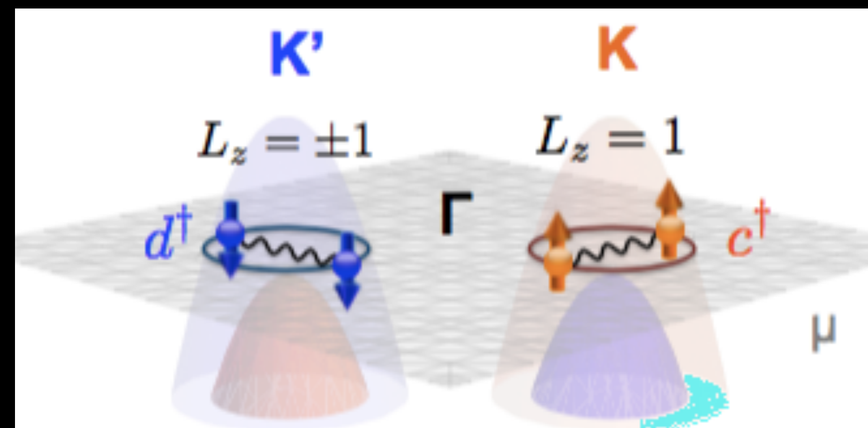
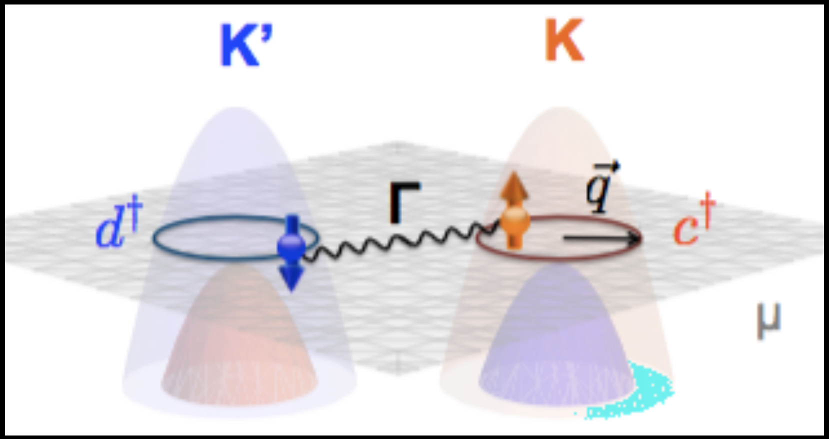


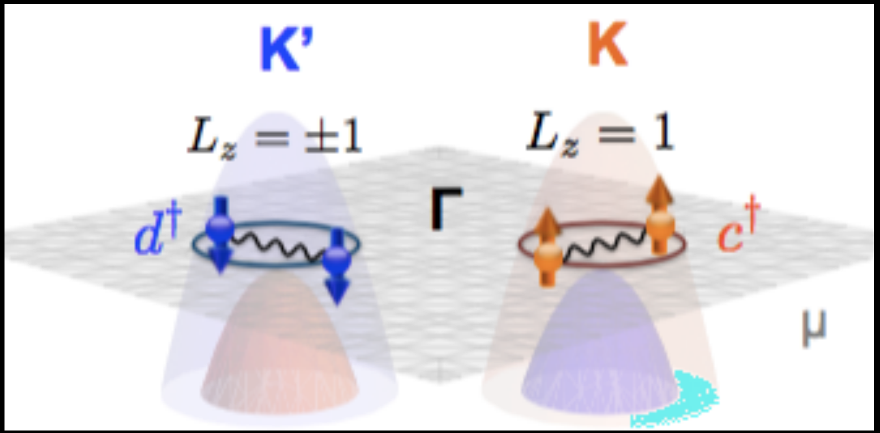
-Modulated

Two possibilities

- Intra-pocket p+ip



- Inter-pocket p'wave

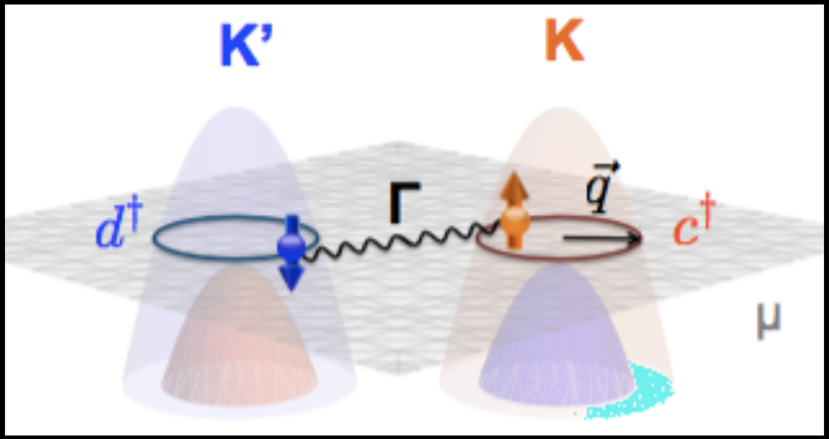


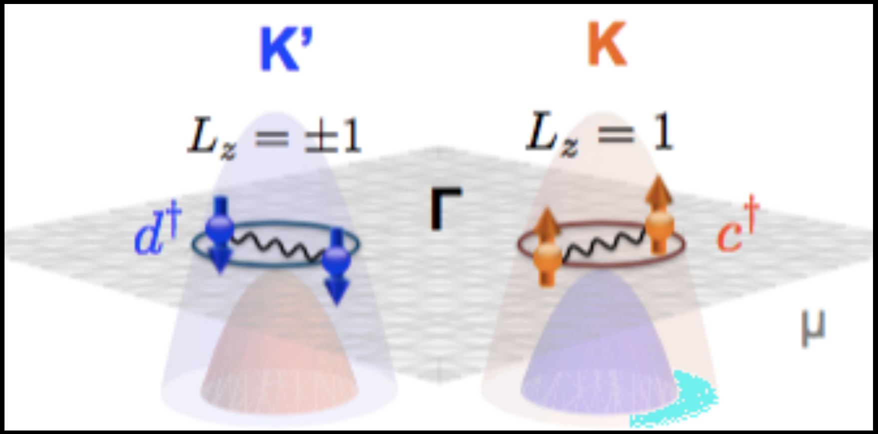
-T-breaking
- C=2

-Modulated

Two possibilities

- Intra-pocket p+ip
- Inter-pocket p'wave



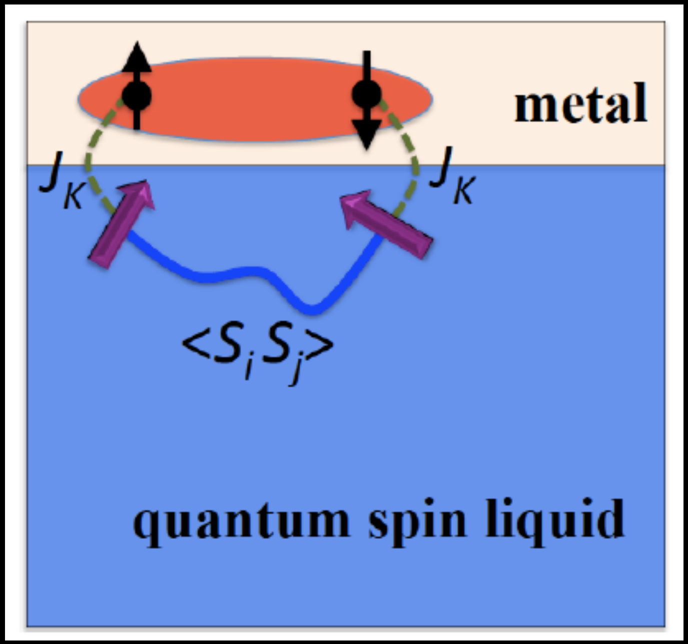


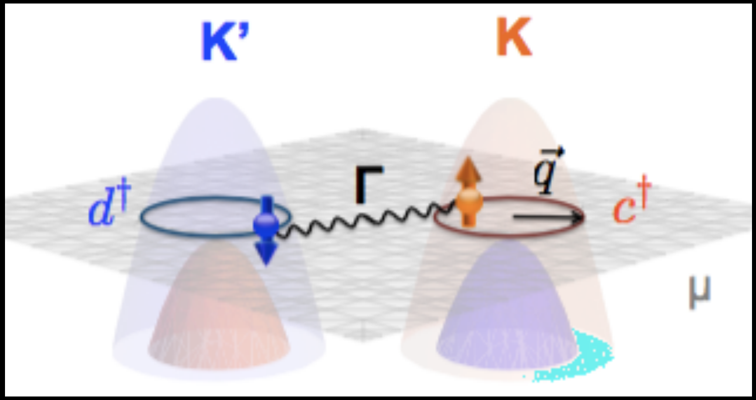
-T-breaking
- C=2

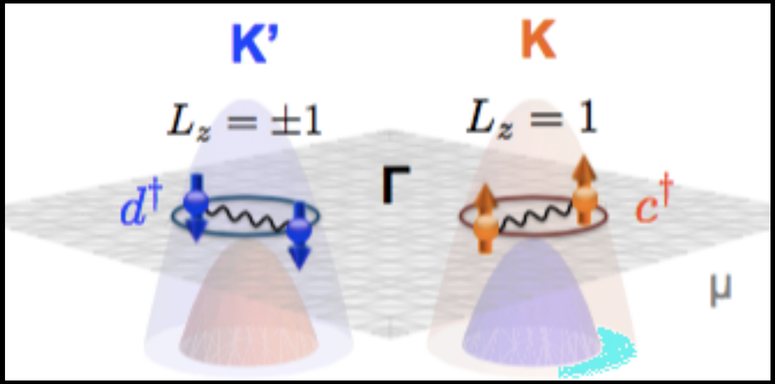
-Modulated
-C=\pm 1 per pocket

Designing 2D topological SC's

- Control interaction
- k-space spin split TMD

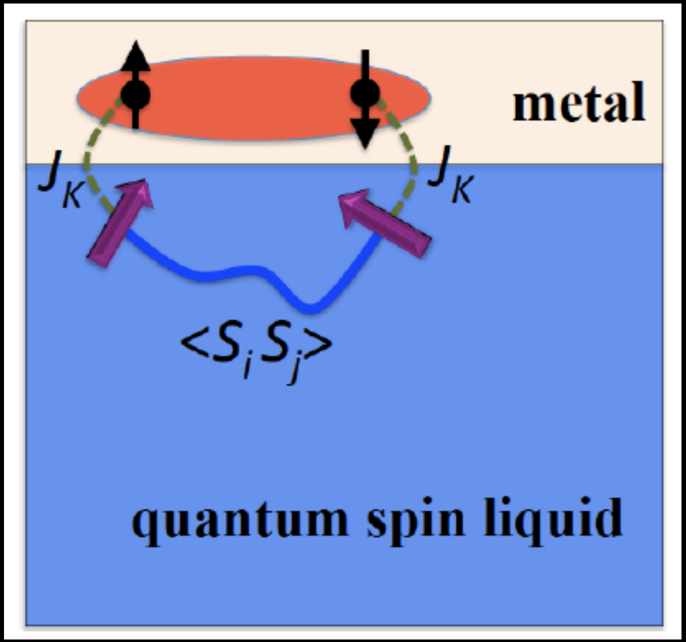






Designing 2D topological SC's

- Control interaction



- k-space spin split TMD