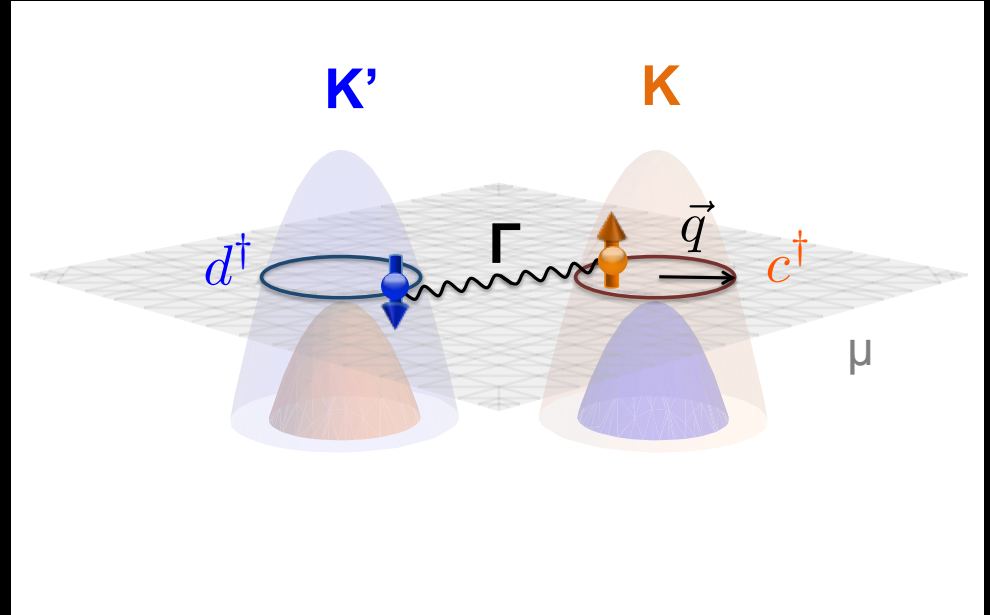
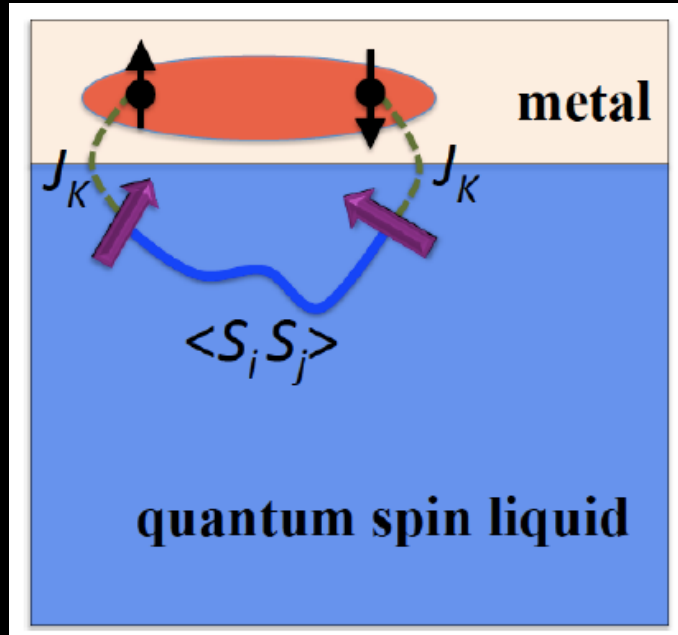


# Designing 2D topological SC's

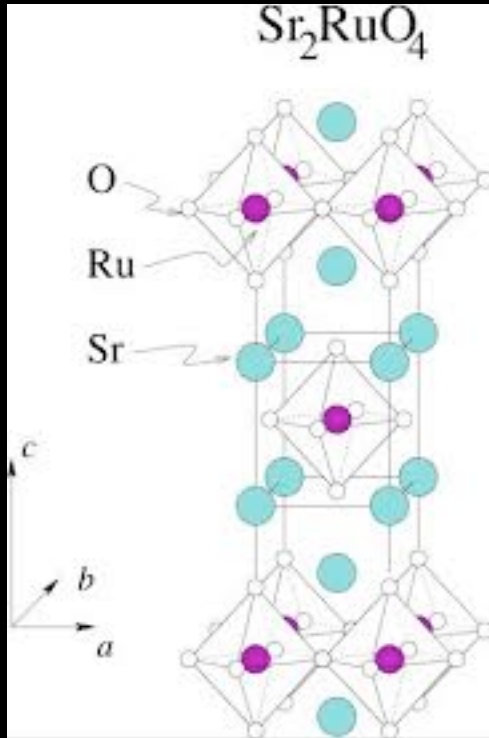


Eun-Ah Kim (Cornell)

Q. Topological Superconductor  
material?

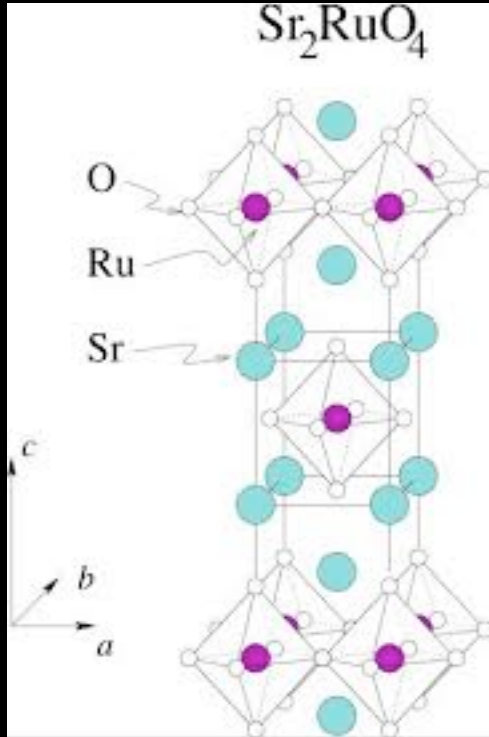
# Q. Topological Superconductor material?

Bulk

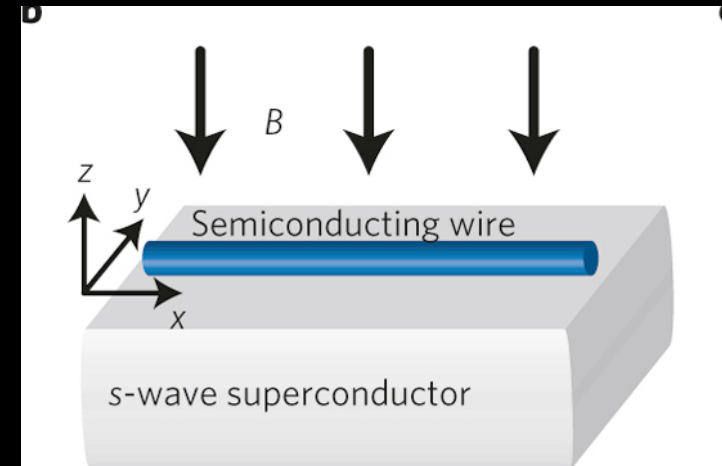


# Q. Topological Superconductor material?

Bulk

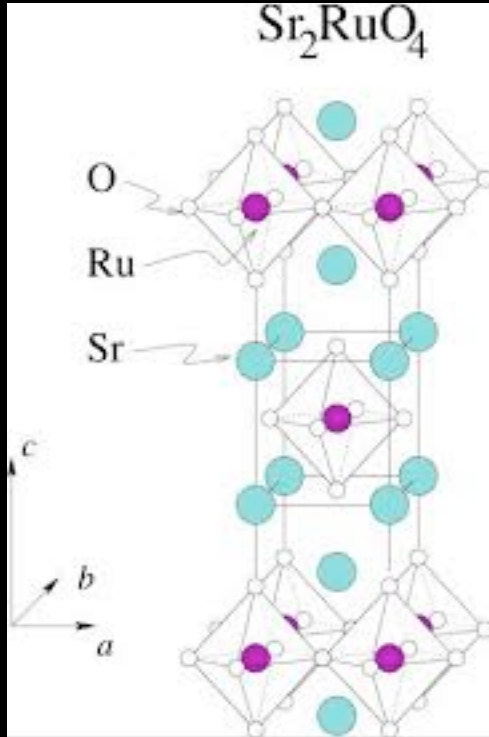


1D proximity

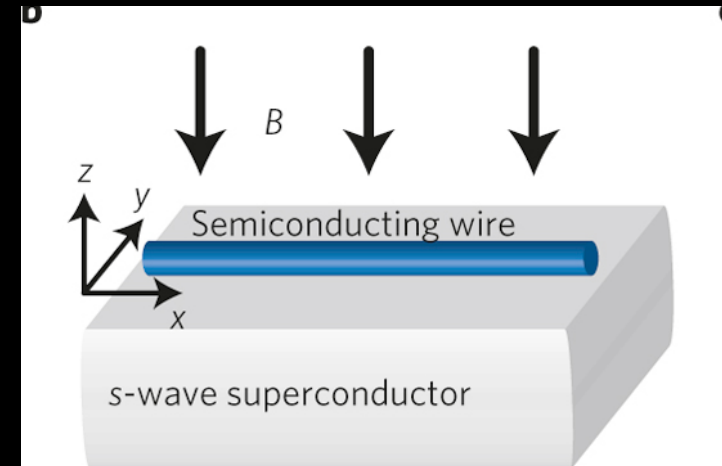


# Q. Topological Superconductor material?

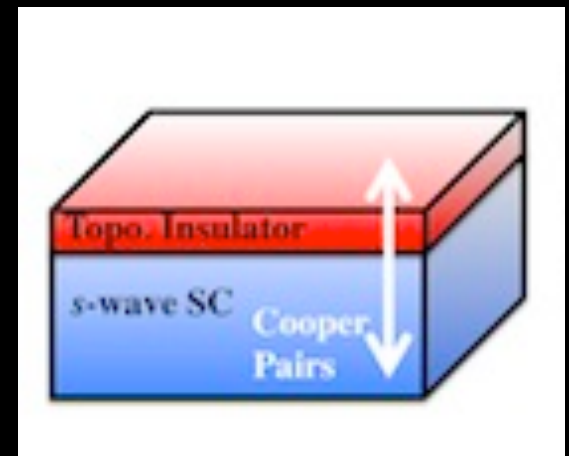
Bulk



1D proximity



2D proximity?



# Designing 2D topological SC's

# Designing 2D topological SC's

- 2D topological SC
  - odd-parity SC of spinless fermions
  - Majorana bound state

# Designing 2D topological SC's

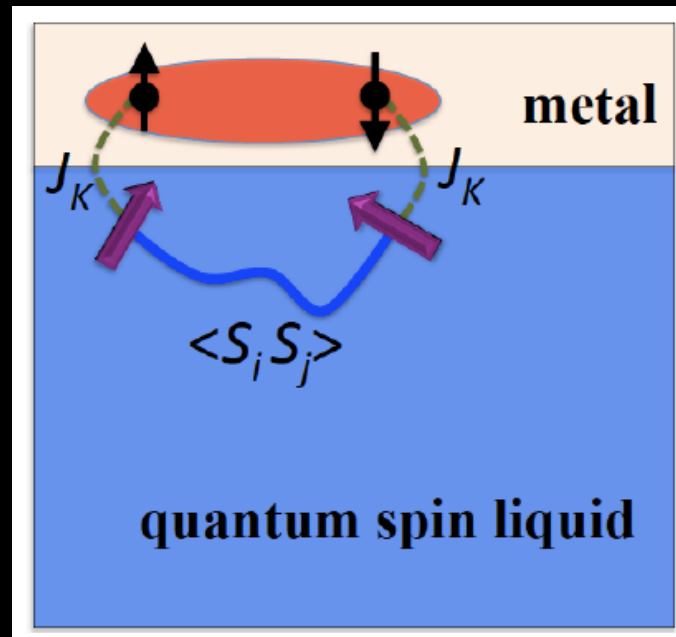
- 2D topological SC
  - odd-parity SC of spinless fermions
  - Majorana bound state
- Strategies:
  - 1) interaction,
  - 2) spinlessness



# Strategy I

- Manipulate **the pairing interaction**:  
target non-phononic mechanism

# Topological Superconductivity in Metal/ Quantum-Spin-Ice Heterostructures



Jian-Huang She, Choonghyun Kim, Craig Fennie,  
Michael Lawler, E-AK (in preparation, 2015)

# Wanted: non-phononic mechanism



P.W.Anderson

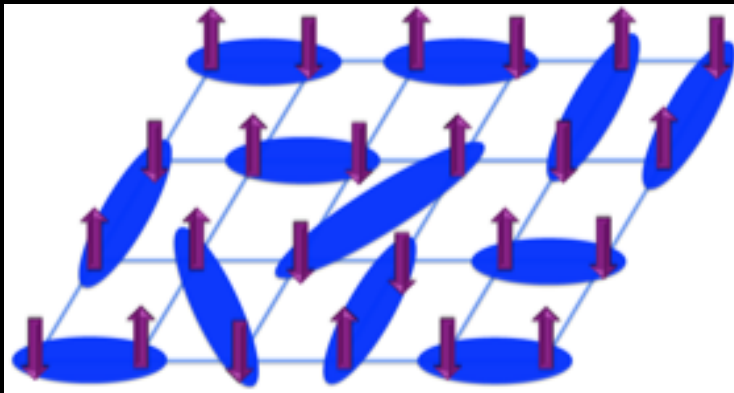
**Dope** a Quantum spin liquid

# Wanted: non-phononic mechanism



P.W.Anderson

**Dope** a Quantum spin liquid

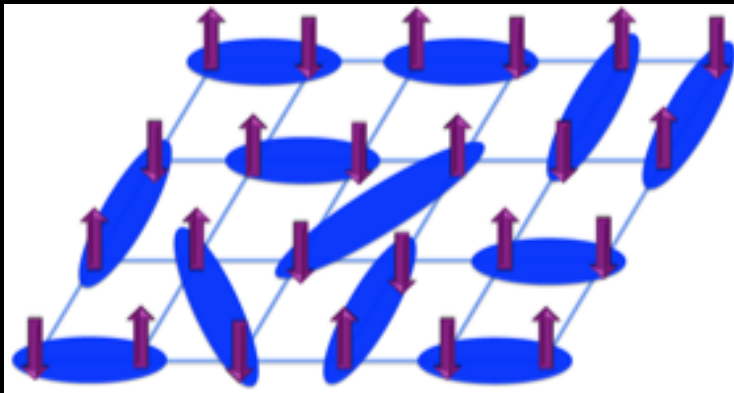


# Wanted: non-phononic mechanism



P.W.Anderson

**Dope** a Quantum spin liquid



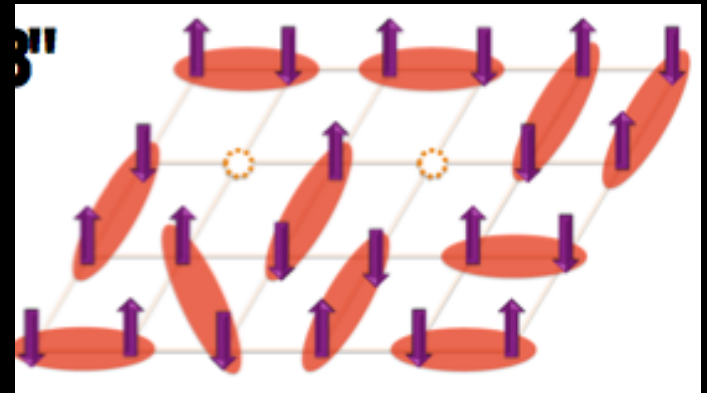
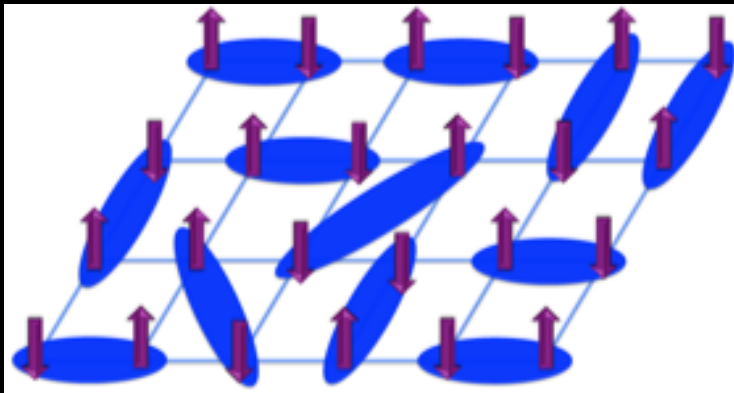
RVB singlet

# Wanted: non-phononic mechanism



P.W.Anderson

**Dope** a Quantum spin liquid



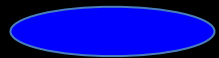
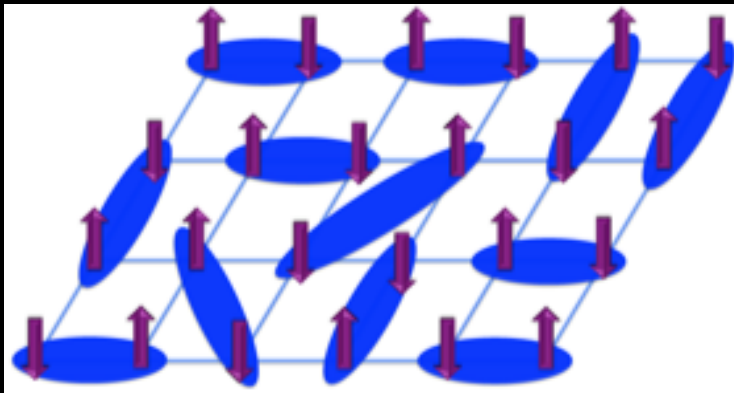
RVB singlet

# Wanted: non-phononic mechanism

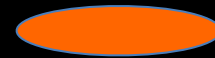
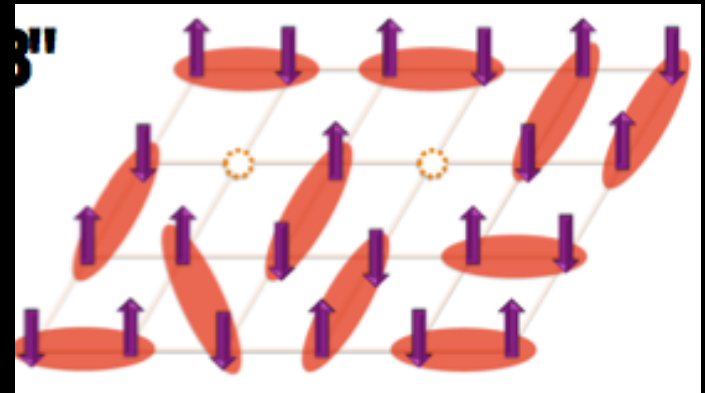


P.W.Anderson

**Dope** a Quantum spin liquid



RVB singlet

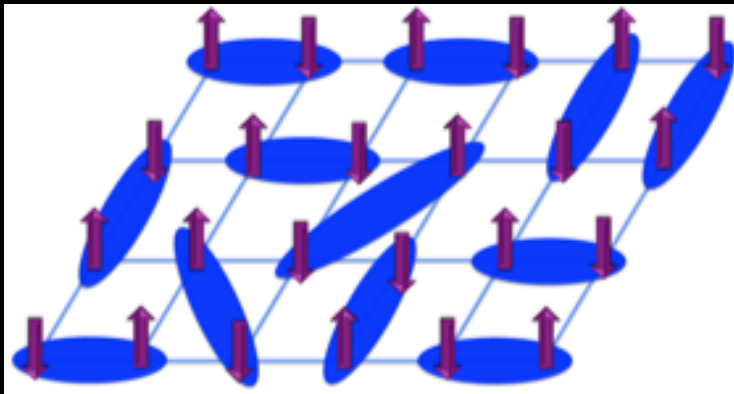


Cooper pair singlet

# Wanted: non-phononic mechanism



Use Quantum spin liquid

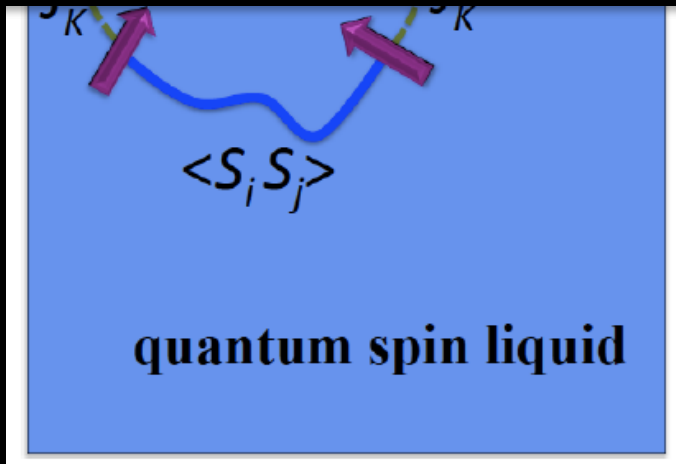




# Wanted: non-phononic mechanism



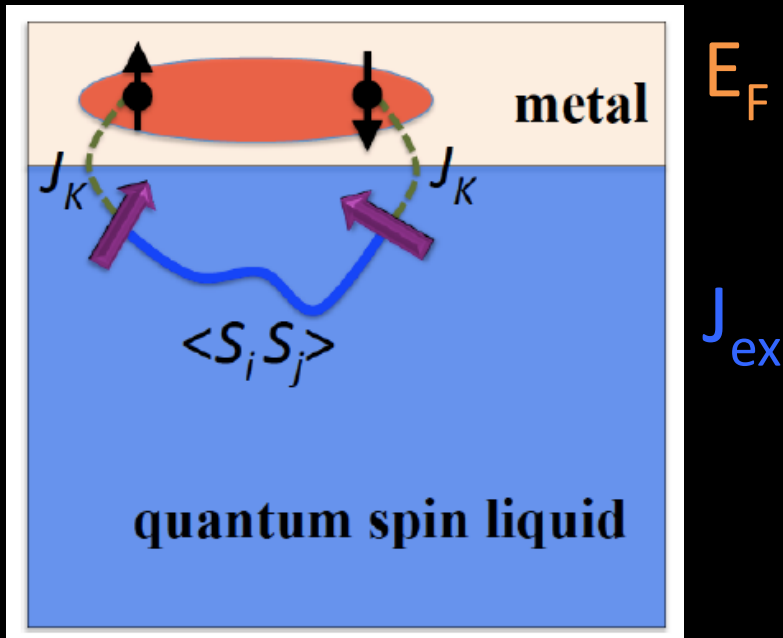
Use Quantum spin liquid



# Wanted: non-phononic mechanism



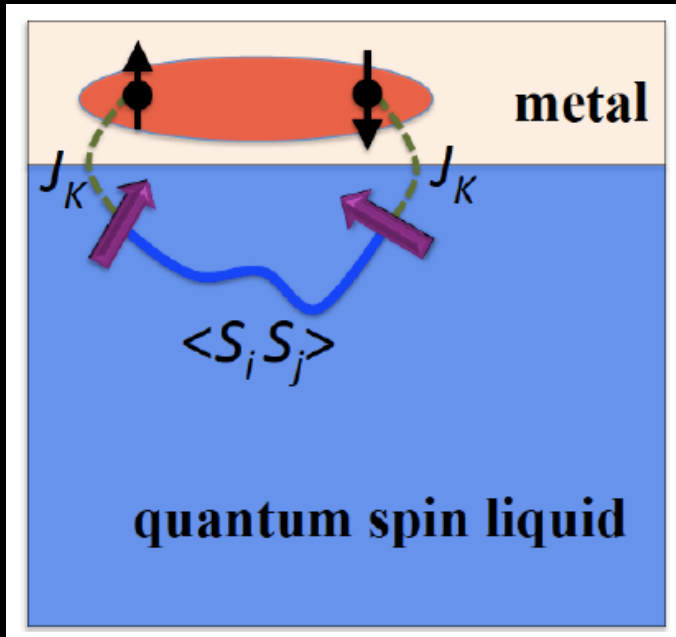
Use Quantum spin liquid



# Wanted: non-phononic mechanism



Use Quantum spin liquid

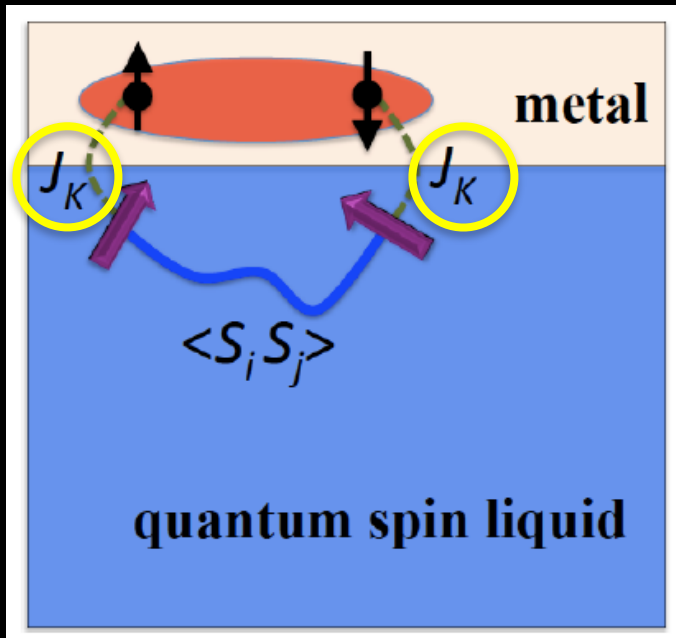


- Characteristic energy scales:

# Wanted: non-phononic mechanism



Use Quantum spin liquid



$E_F$

$E_F, J_{ex}, J_K$

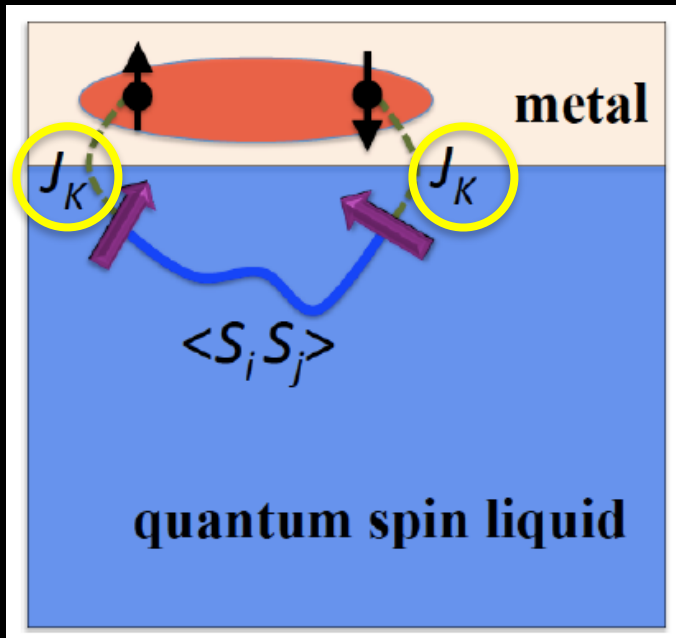
$J_{ex}$

- Characteristic energy scales:

# Wanted: non-phononic mechanism



Use Quantum spin liquid



$E_F$

$E_F, J_{ex}, J_K$

$J_{ex}$

- Characteristic energy scales:

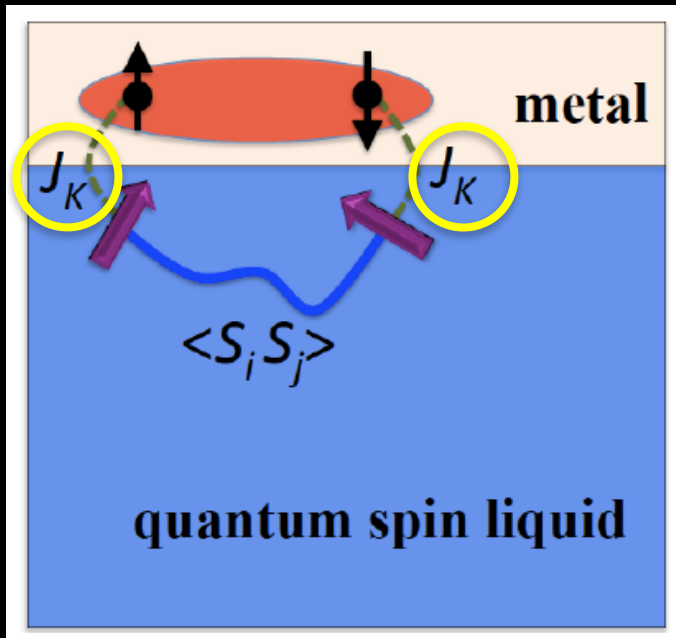
- Perturbative limit:

$$J_K / E_F \ll 1$$

# Wanted: non-phononic mechanism



Use Quantum spin liquid



- Characteristic energy scales:

$$E_F, J_{ex}, J_K$$

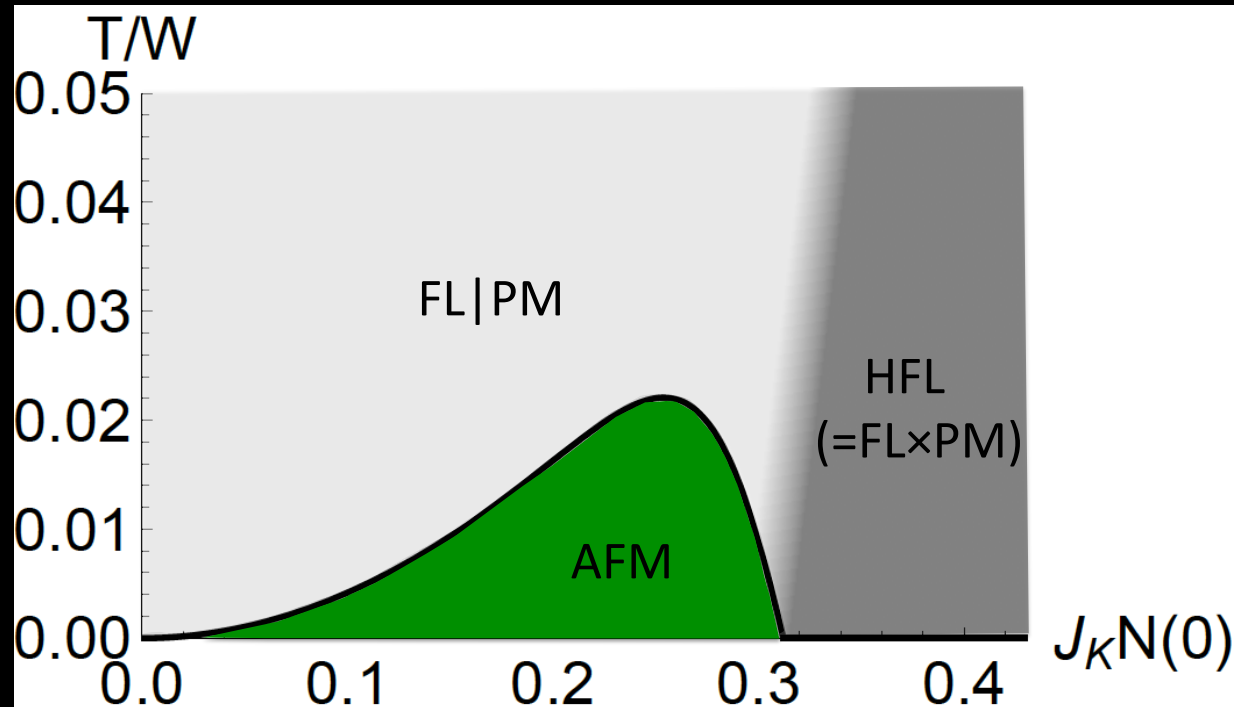
- Perturbative limit:

$$J_K / E_F \ll 1$$

- Spin-fermion model

# Spin-fermion model for $J_{\text{ex}}=0$

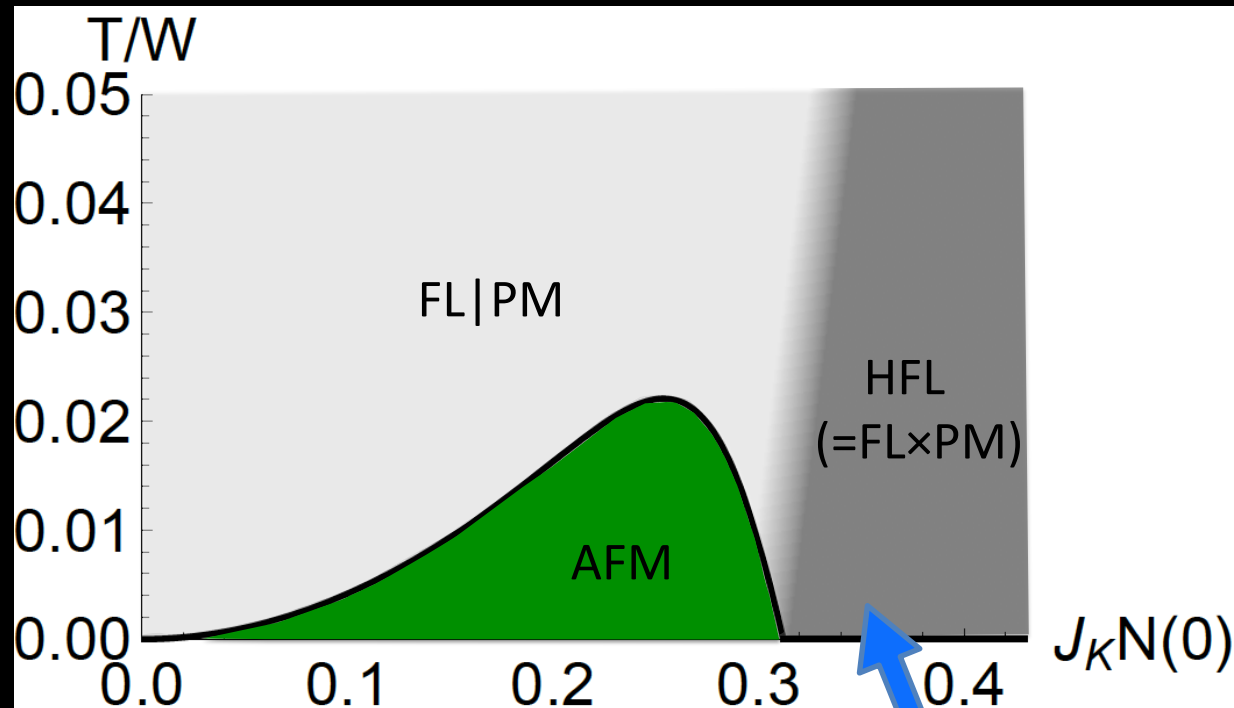
# Spin-fermion model for $J_{\text{ex}}=0$



Doniach (1977)



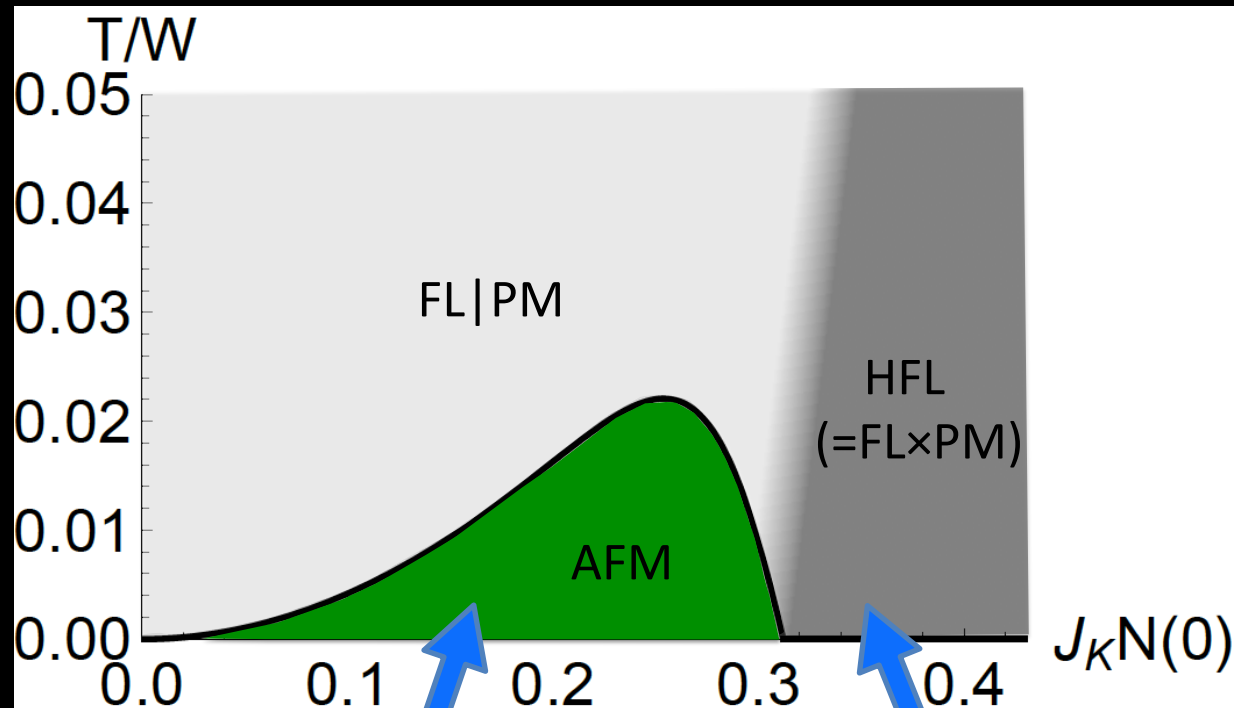
# Spin-fermion model for $J_{\text{ex}}=0$



Kondo-Singlet

Doniach (1977)

# Spin-fermion model for $J_{\text{ex}}=0$



RKKY interaction

Kondo-Singlet

Doniach (1977)

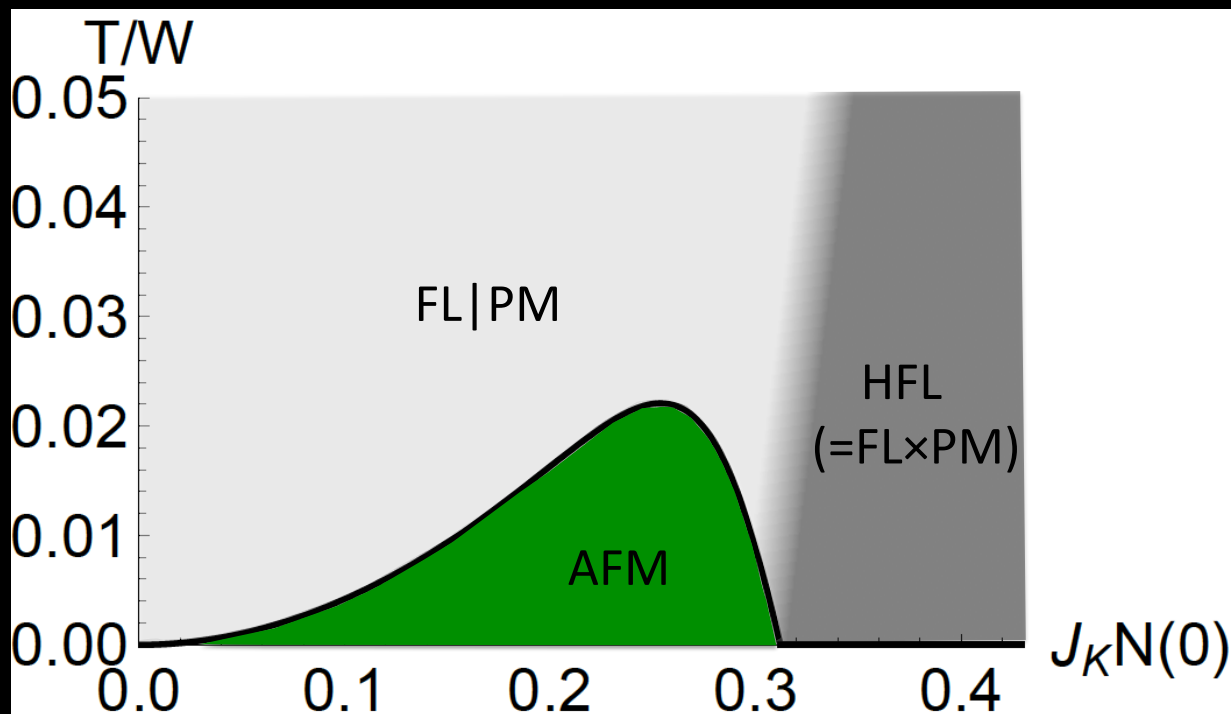
# Spin-fermion model for $J_{\text{ex}}$ + Frustration

# Spin-fermion model for $J_{\text{ex}}$ + Frustration

For  $J_{\text{RKKY}} \sim J_{\text{K}}^2 N(0) < J_{\text{ex}}$  AFM order suppressed.

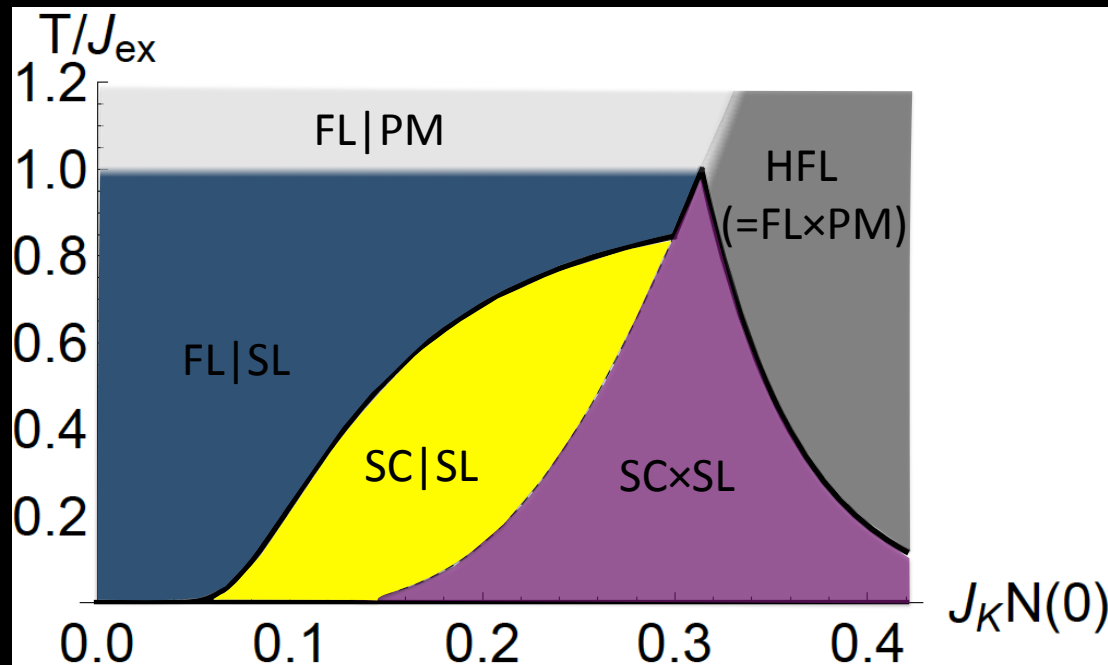
# Spin-fermion model for $J_{\text{ex}}$ + Frustration

For  $J_{\text{RKKY}} \sim J_K^2 N(0) < J_{\text{ex}}$  AFM order suppressed.



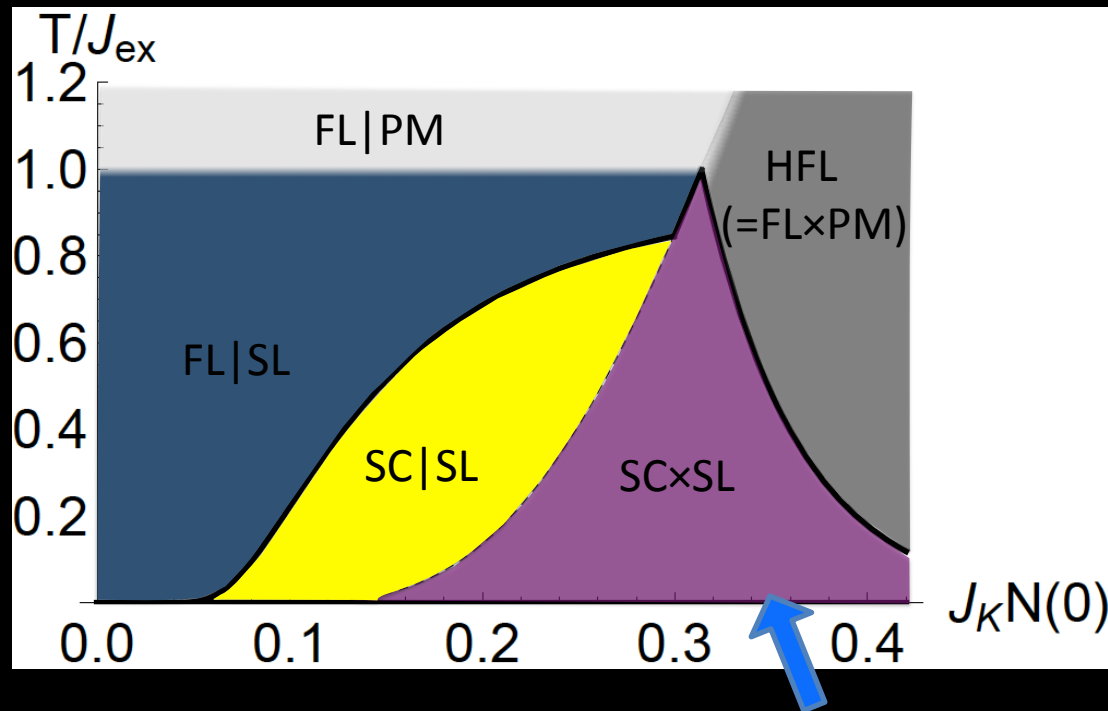
# Spin-fermion model for $J_{\text{ex}}$ + Frustration

For  $J_{\text{RKKY}} \sim J_K^2 N(0) < J_{\text{ex}}$  AFM order suppressed.



# Spin-fermion model for $J_{\text{ex}}$ + Frustration

For  $J_{\text{RKKY}} \sim J_K^2 N(0) < J_{\text{ex}}$  AFM order suppressed.



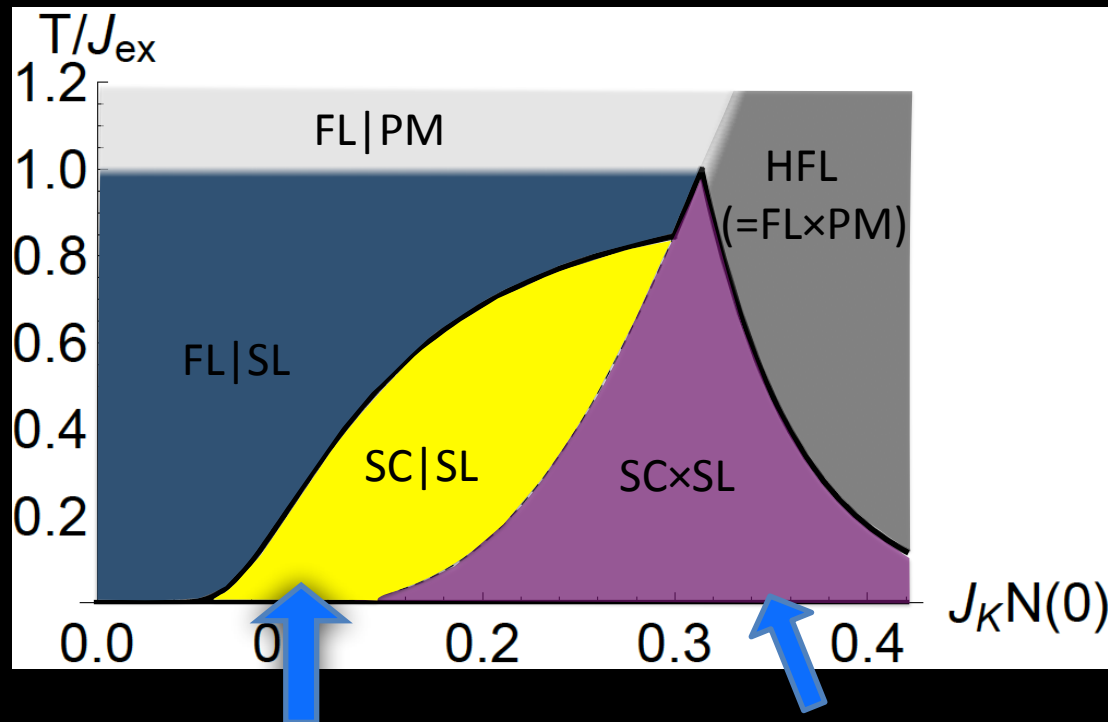
Kondo-Singlet + RVB singlet  
+Cooper pair singlet

Coleman & Andrei (XXXX)

Senthil, Vojta, Sachdev (XXXX)

# Spin-fermion model for $J_{\text{ex}}$ + Frustration

For  $J_{\text{RKKY}} \sim J_K^2 N(0) < J_{\text{ex}}$  AFM order suppressed.



Superconductor  
“riding” on QSL

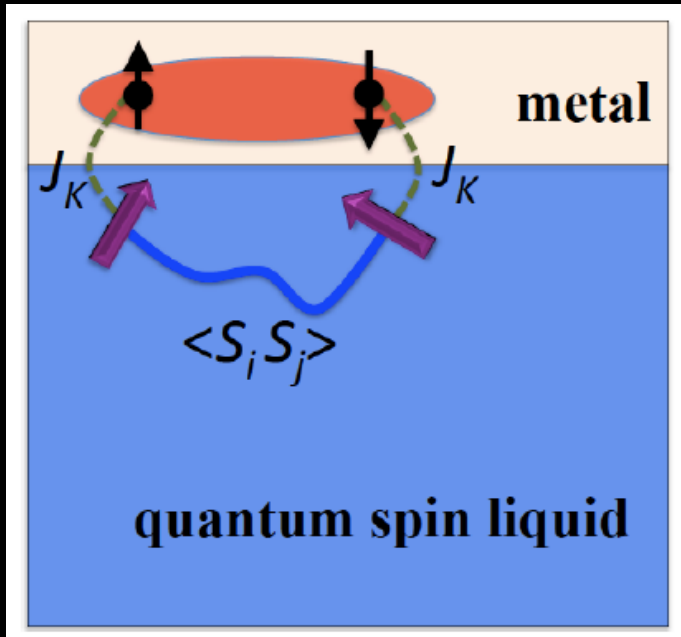
Kondo-Singlet + RVB singlet  
+Cooper pair singlet

Coleman & Andrei (XXXX)

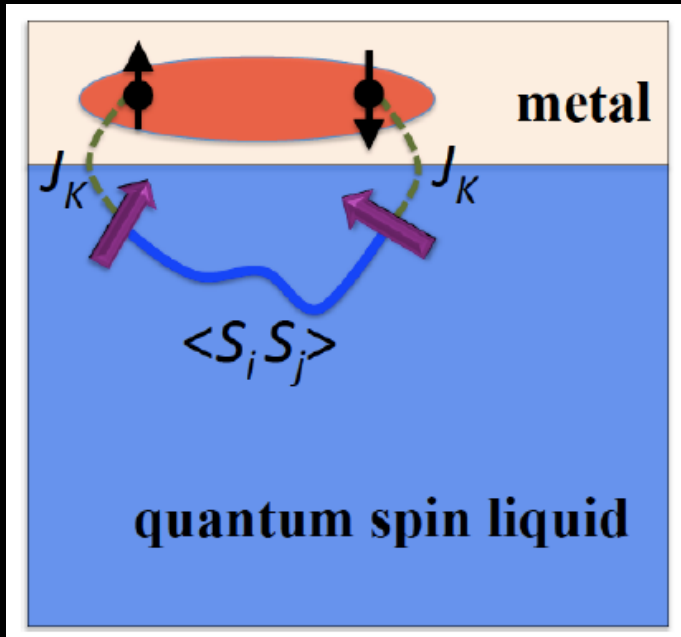
Senthil, Vojta, Sachdev (XXXX)



# How to predictively materialize SC|QSL ?

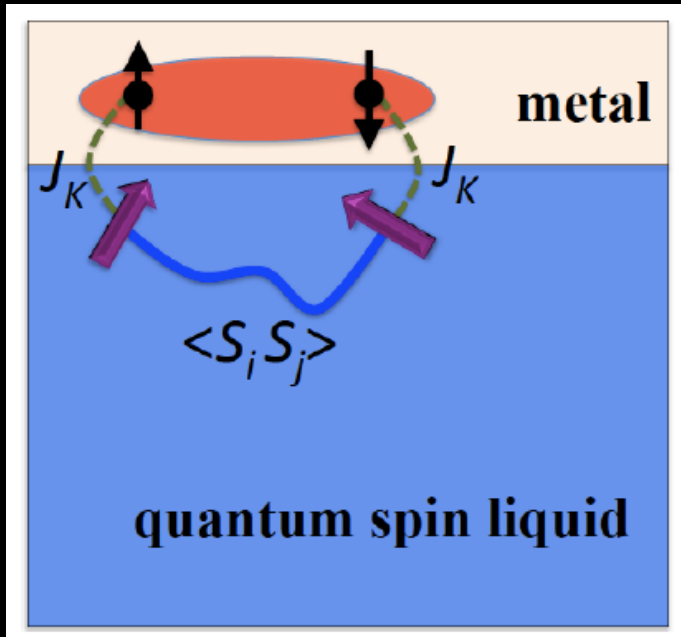


# How to predictively materialize SC|QSL ?



**Simple isotropic metal**

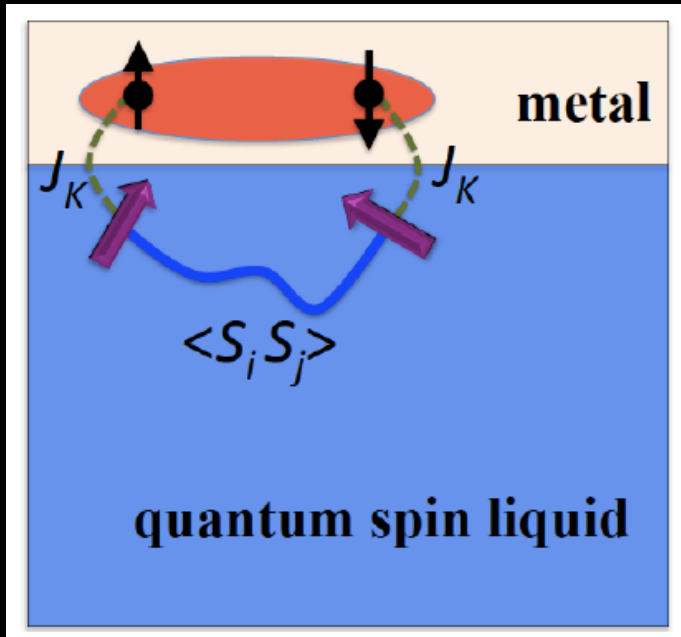
# How to predictively materialize SC|QSL ?



**Simple isotropic metal**

1.  $\langle S \rangle = 0$

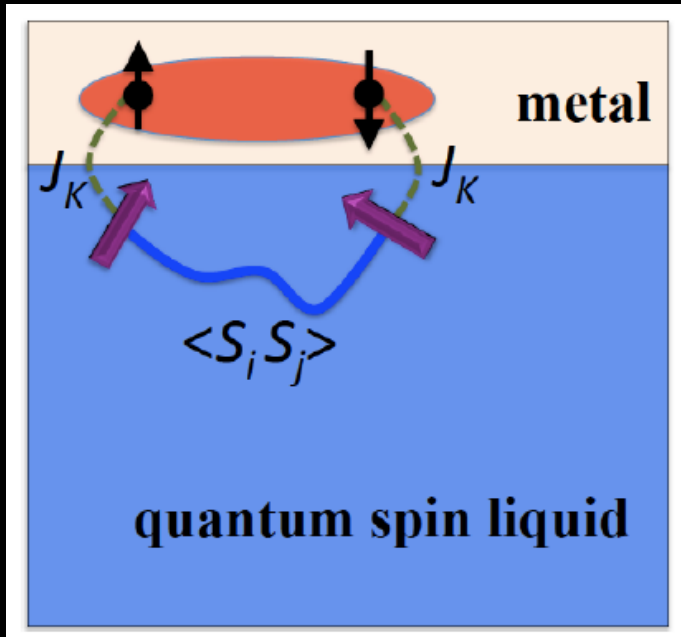
# How to predictively materialize SC|QSL ?



## Simple isotropic metal

1.  $\langle S \rangle = 0$
2. Dynamic spin fluctuation  $\langle S_i S_j \rangle$

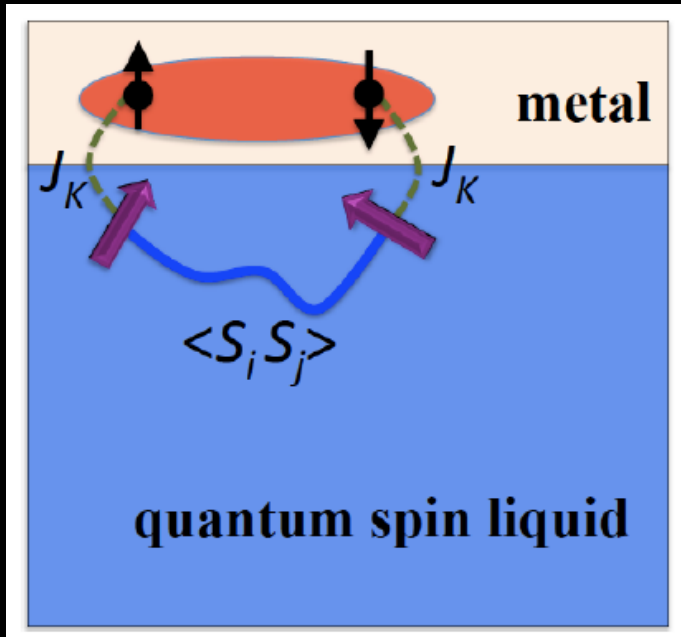
# How to predictively materialize SC|QSL ?



## Simple isotropic metal

1.  $\langle S \rangle = 0$
2. Dynamic spin fluctuation  $\langle S_i S_j \rangle$
3. Gapped spectrum

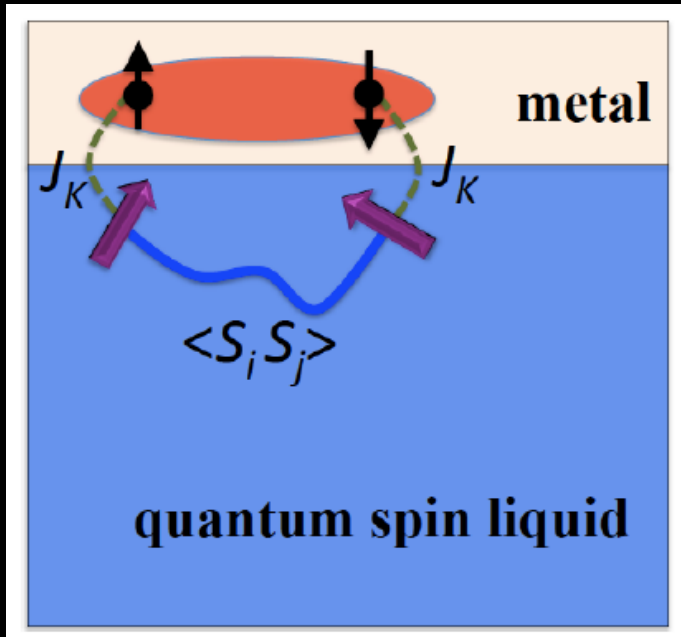
# How to predictively materialize SC|QSL ?



## Simple isotropic metal

1.  $\langle S \rangle = 0$
2. Dynamic spin fluctuation  $\langle S_i S_j \rangle$
3. Gapped spectrum
4. “Simple”

# How to predictively materialize SC|QSL ?



## Simple isotropic metal

1.  $\langle S \rangle = 0$
2. Dynamic spin fluctuation  $\langle S_i S_j \rangle$
3. Gapped spectrum
4. "Simple"

➡ Quantum Spin Ice

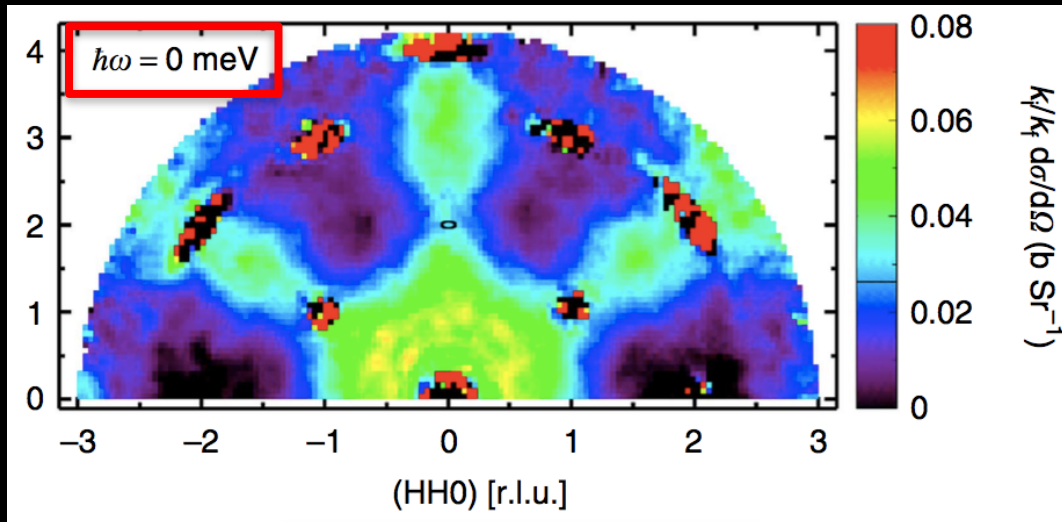
# Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>



# Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

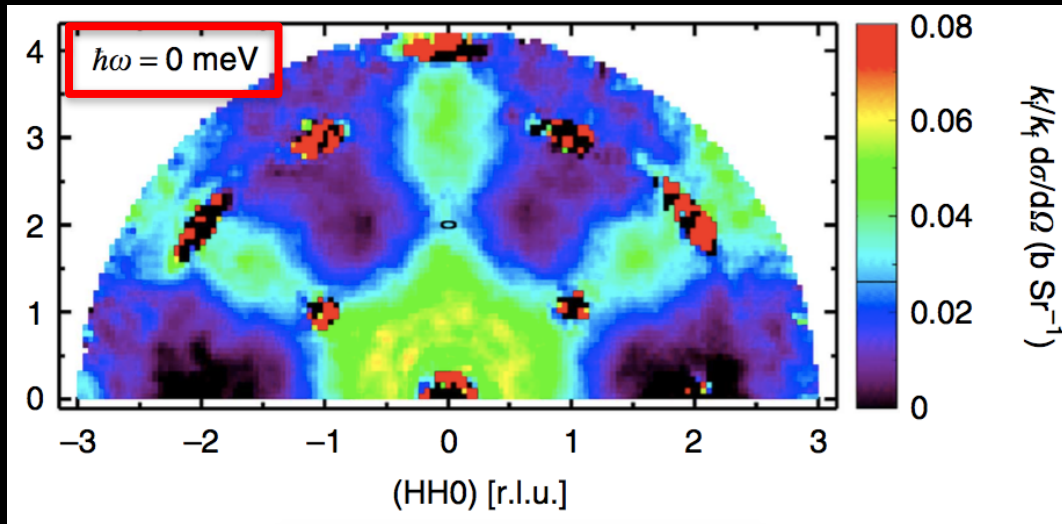
K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>



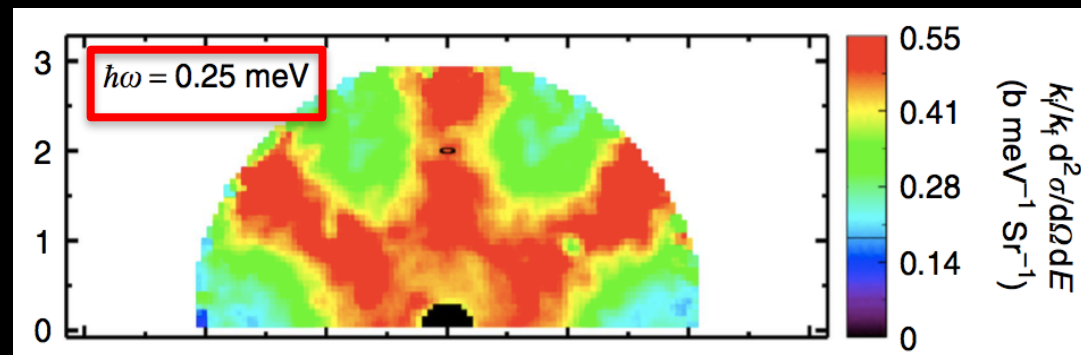
- Elastic neutron: pinch points (spin-ice like)

# Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>



- Elastic neutron: pinch points (spin-ice like)



- Inelastic neutron: over 90% weight

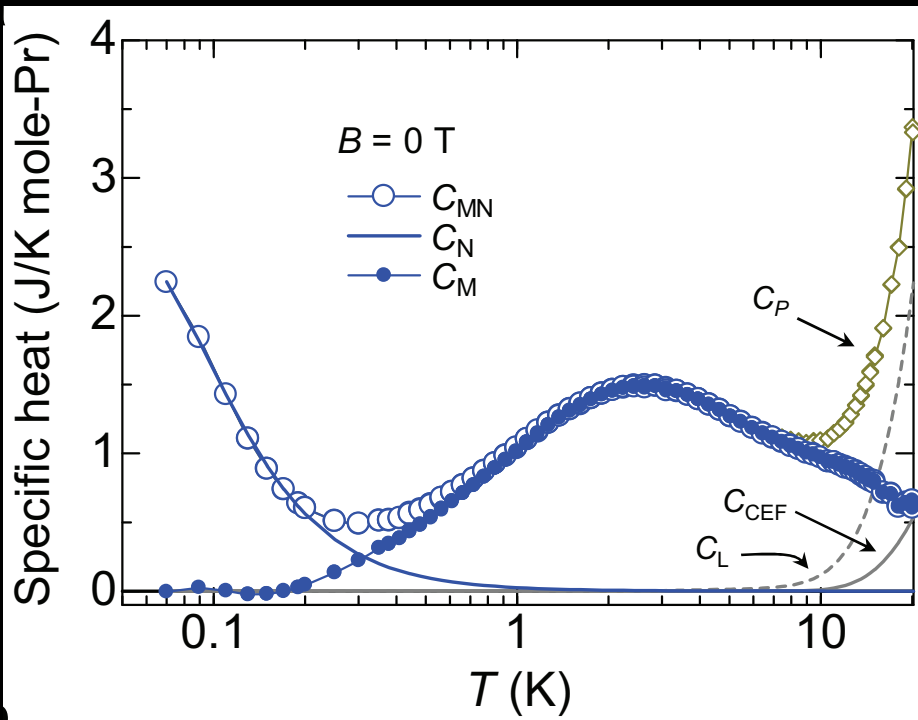
# Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>

- No order down to 20mK

# Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

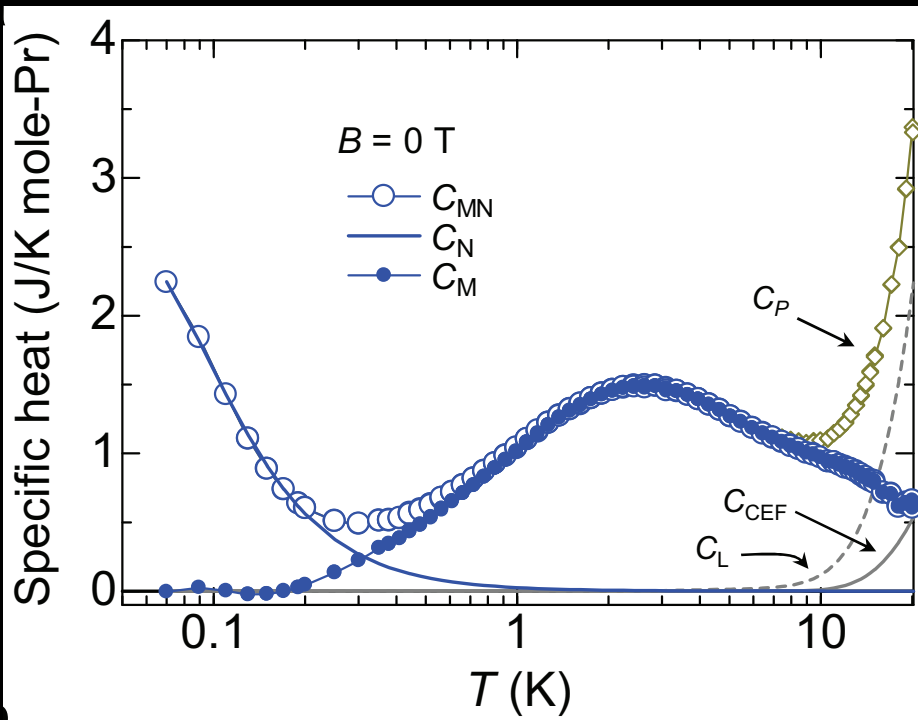
K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>



- No order down to 20mK
- Gapped quantum paramagnet

# Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

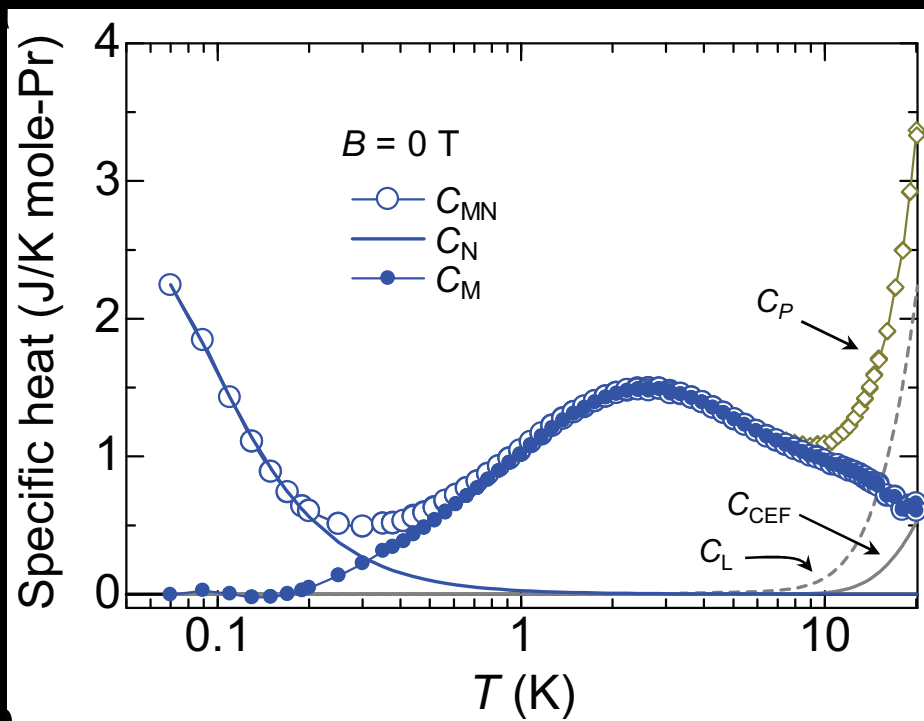
K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>



- No order down to 20mK
- Gapped quantum paramagnet  $\omega_s=0.17\text{meV}$

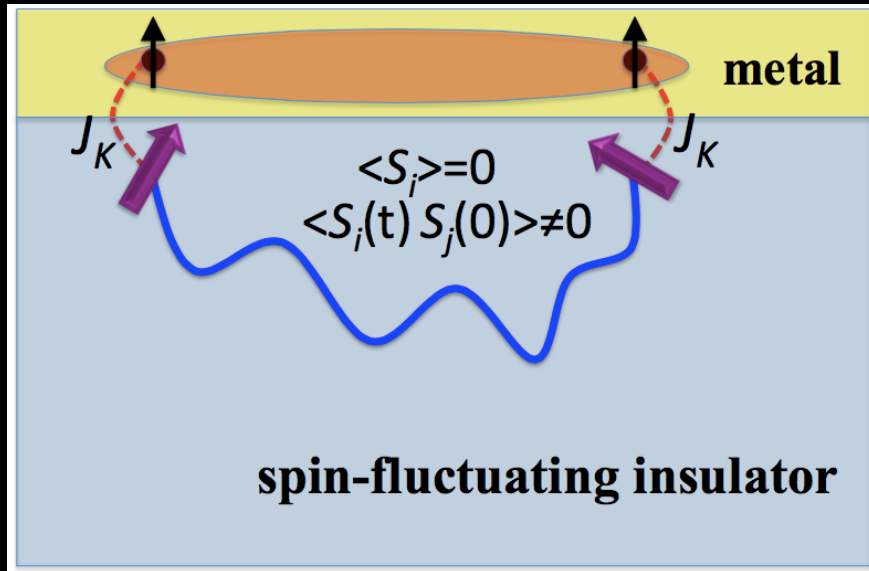
# Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

K. Kimura<sup>1</sup>, S. Nakatsuji<sup>1,2</sup>, J.-J. Wen<sup>3</sup>, C. Broholm<sup>3,4,5</sup>, M.B. Stone<sup>5</sup>, E. Nishibori<sup>6</sup> & H. Sawa<sup>6</sup>

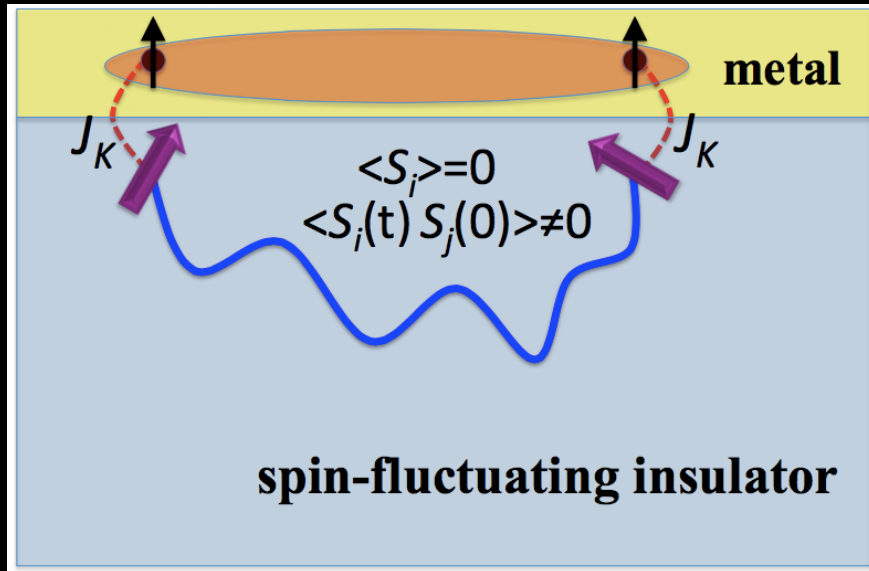


- No order down to 20mK
- Gapped quantum paramagnet  $\omega_s=0.17\text{meV}$
- Inelastic spectra peaked at  $Q=0$

# Effective Continuum Theory



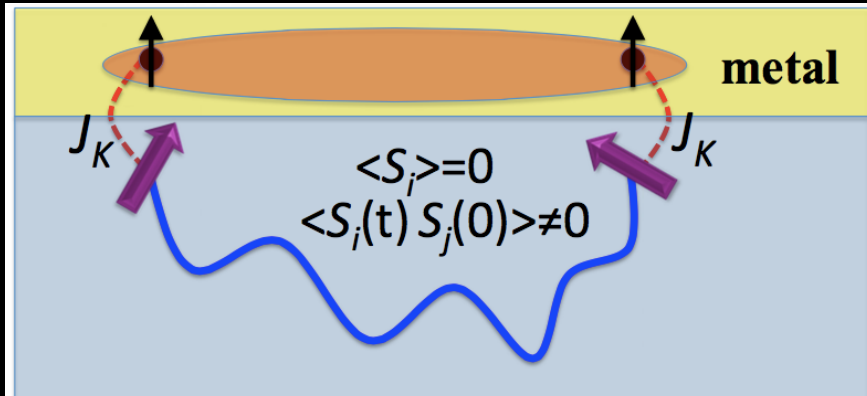
# Effective Continuum Theory



$$H_c = \sum_{\mathbf{k}\alpha} \left( \frac{\hbar^2 k^2}{2m} - E_F \right) \psi_{\alpha}^{\dagger}(\mathbf{k}) \psi_{\alpha}(\mathbf{k})$$



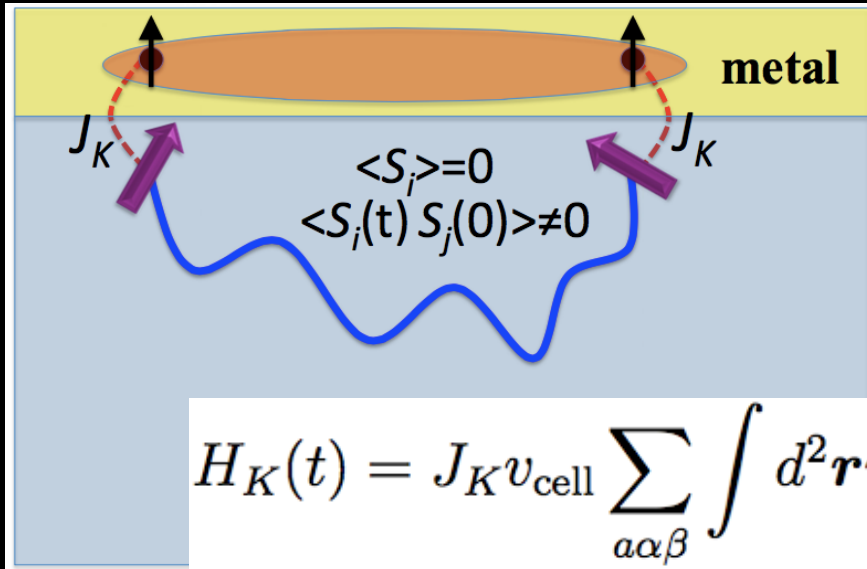
# Effective Continuum Theory



$$H_c = \sum_{\mathbf{k}\alpha} \left( \frac{\hbar^2 k^2}{2m} - E_F \right) \psi_{\alpha}^{\dagger}(\mathbf{k}) \psi_{\alpha}(\mathbf{k})$$

$$H_K(t) = J_K v_{\text{cell}} \sum_{\alpha\beta} \int d^2\mathbf{r} \psi_{\alpha}^{\dagger}(\mathbf{r}) \sigma_{\alpha\beta}^a \psi_{\beta}(\mathbf{r}) S_a(\mathbf{r}_{\perp} = \mathbf{r}, z = 0, t)$$

# Effective Continuum Theory

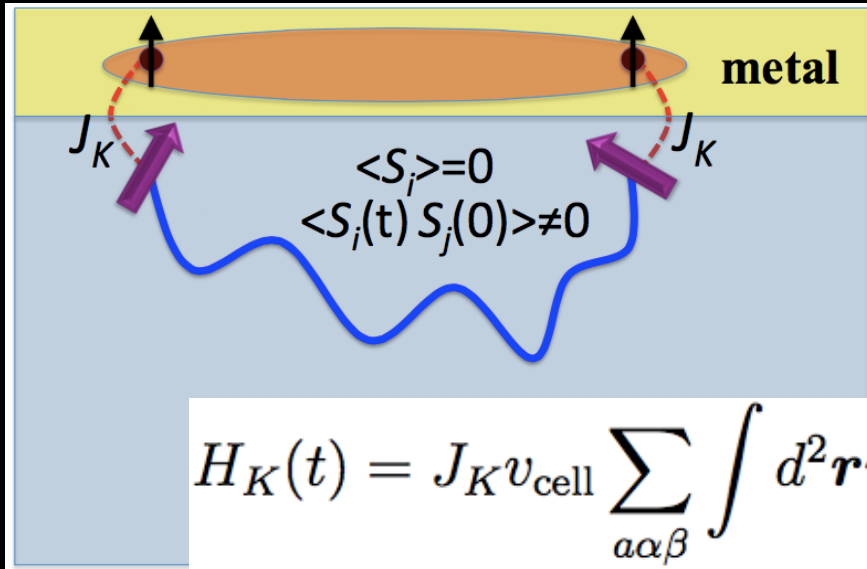


$$H_c = \sum_{\mathbf{k}\alpha} \left( \frac{\hbar^2 k^2}{2m} - E_F \right) \psi_{\alpha}^{\dagger}(\mathbf{k}) \psi_{\alpha}(\mathbf{k})$$

$$H_K(t) = J_K v_{\text{cell}} \sum_{\alpha\beta} \int d^2\mathbf{r} \psi_{\alpha}^{\dagger}(\mathbf{r}) \sigma_{\alpha\beta}^a \psi_{\beta}(\mathbf{r}) S_a(\mathbf{r}_{\perp} = \mathbf{r}, z = 0, t)$$

- Integrate out spins >> Effective e-e interaction

# Effective Continuum Theory



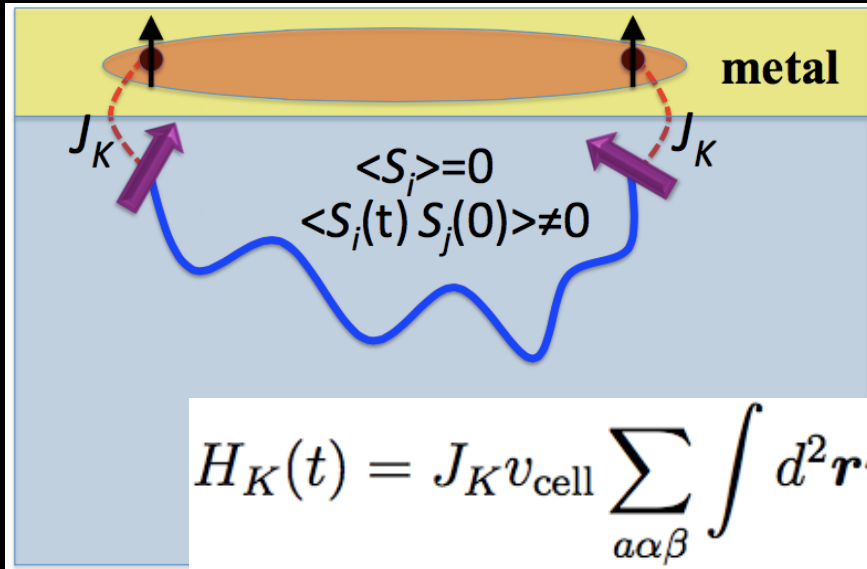
$$H_c = \sum_{\mathbf{k}\alpha} \left( \frac{\hbar^2 k^2}{2m} - E_F \right) \psi_{\alpha}^{\dagger}(\mathbf{k}) \psi_{\alpha}(\mathbf{k})$$

$$H_K(t) = J_K v_{\text{cell}} \sum_{a\alpha\beta} \int d^2\mathbf{r} \psi_{\alpha}^{\dagger}(\mathbf{r}) \sigma_{\alpha\beta}^a \psi_{\beta}(\mathbf{r}) S_a(\mathbf{r}_{\perp} = \mathbf{r}, z = 0, t)$$

- Integrate out spins >> Effective e-e interaction

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

# Effective Continuum Theory



$$H_c = \sum_{\mathbf{k}\alpha} \left( \frac{\hbar^2 k^2}{2m} - E_F \right) \psi_{\alpha}^{\dagger}(\mathbf{k}) \psi_{\alpha}(\mathbf{k})$$

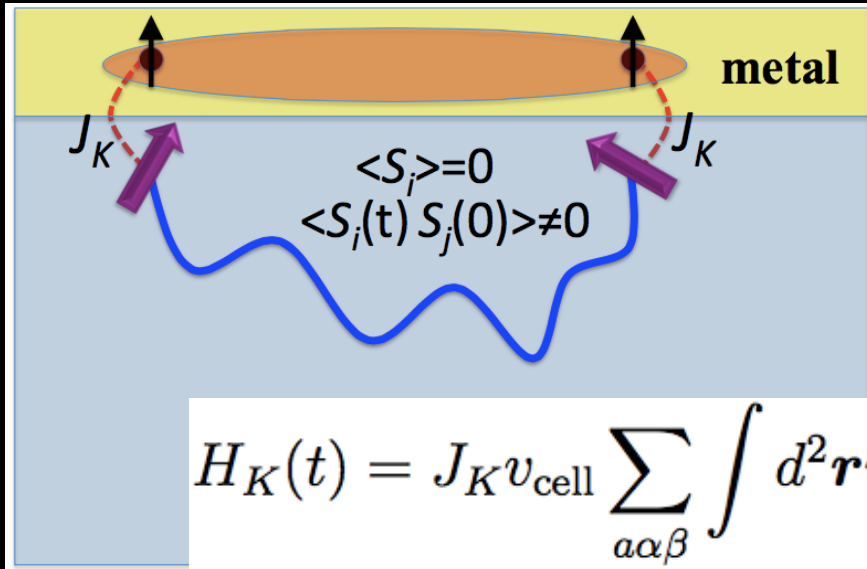
$$H_K(t) = J_K v_{\text{cell}} \sum_{a\alpha\beta} \int d^2\mathbf{r} \psi_{\alpha}^{\dagger}(\mathbf{r}) \sigma_{\alpha\beta}^a \psi_{\beta}(\mathbf{r}) S_a(\mathbf{r}_{\perp} = \mathbf{r}, z = 0, t)$$

- Integrate out spins >> Effective e-e interaction

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

$$s_a(\mathbf{r}, t) = \sum_{\alpha\beta} \psi_{\alpha}^{\dagger}(\mathbf{r}, t) \sigma_{\alpha\beta}^a \psi_{\beta}(\mathbf{r}, t)$$

# Effective Continuum Theory



$$H_c = \sum_{\mathbf{k}\alpha} \left( \frac{\hbar^2 k^2}{2m} - E_F \right) \psi_{\alpha}^{\dagger}(\mathbf{k}) \psi_{\alpha}(\mathbf{k})$$

$$H_K(t) = J_K v_{\text{cell}} \sum_{a\alpha\beta} \int d^2\mathbf{r} \psi_{\alpha}^{\dagger}(\mathbf{r}) \sigma_{\alpha\beta}^a \psi_{\beta}(\mathbf{r}) S_a(\mathbf{r}_{\perp} = \mathbf{r}, z = 0, t)$$

- Integrate out spins >> Effective e-e interaction

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

$$s_a(\mathbf{r}, t) = \sum_{\alpha\beta} \psi_{\alpha}^{\dagger}(\mathbf{r}, t) \sigma_{\alpha\beta}^a \psi_{\beta}(\mathbf{r}, t)$$

# Dealing with interacting electrons?

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

# Dealing with interacting electrons?

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

- Separation of scale:  $\omega_s/E_F \ll 1$   
     $\Rightarrow$  “Migdal theorem”

# Dealing with interacting electrons?

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

- Separation of scale:  $\omega_s/E_F \ll 1$

➡ “Migdal theorem”

- Dimensionless ratio:  $\lambda \sim N(0)V \sim J_K^2 N(0)/J_{\text{ex}} < 1$



# Dealing with interacting electrons?

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

- Separation of scale:  $\omega_s/E_F \ll 1$

➔ “Migdal theorem”

- Dimensionless ratio:  $\lambda \sim N(0)V \sim J_K^2 N(0)/J_{\text{ex}} < 1$

- Full problem  $\approx$   
solving the BCS mean-field theory

# Dealing with interacting electrons?

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

- Separation of scale:  $\omega_s/E_F \ll 1$

➔ “Migdal theorem”

- Dimensionless ratio:  $\lambda \sim N(0)V \sim J_K^2 N(0)/J_{\text{ex}} < 1$

- Full problem  $\approx$

solving the BCS mean-field theory

$$T_c \sim \omega_s e^{-1/\lambda}$$

# Dominant Pairing Channel

# Dominant Pairing Channel

- Energy integrated spin structure factor

# Dominant Pairing Channel

- Energy integrated spin structure factor

$$S_{ab}(\mathbf{q}) = \delta_{ab} - \left(1 - \frac{1}{1+q^2\xi^2}\right) \frac{q_a q_b}{q^2}$$

# Dominant Pairing Channel

- Energy integrated spin structure factor

$$S_{ab}(\mathbf{q}) = \delta_{ab} - \left(1 - \frac{1}{1+q^2\xi^2}\right) \frac{q_a q_b}{q^2}$$

1.  $SO(3) \times SU(2)$  reduced to  $U(1)$

# Dominant Pairing Channel

- Energy integrated spin structure factor

$$S_{ab}(\mathbf{q}) = \delta_{ab} - \left(1 - \frac{1}{1+q^2\xi^2}\right) \frac{q_a q_b}{q^2}$$

1.  $SO(3) \times SU(2)$  reduced to  $U(1)$
2. Quantum #'s:  $J_z = L_z + S_z$  & Parity

# Dominant Pairing Channel

- Energy integrated spin structure factor

$$S_{ab}(\mathbf{q}) = \delta_{ab} - \left(1 - \frac{1}{1+q^2\xi^2}\right) \frac{q_a q_b}{q^2}$$

1. SO(3)xSU(2) reduced to U(1)
2. Quantum #'s:  $J_z=L_z+S_z$  & Parity
3. Resulting interaction **suppresses even-parity states**



# Leading channels

# Leading channels

$$\hat{\Delta}_{j_z=0}^{(-)} \sim \begin{pmatrix} k_x - ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix}$$

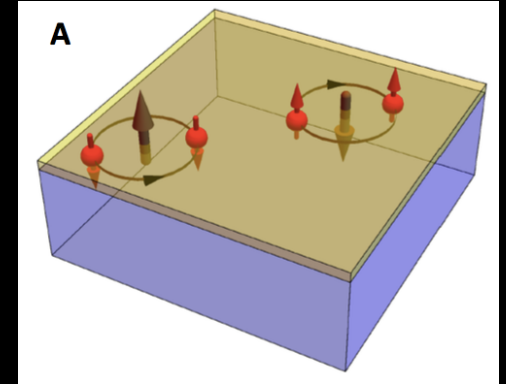
# Leading channels

$$\hat{\Delta}_{j_z=0}^{(-)} \sim \begin{pmatrix} k_x - ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix} \sim \text{He3-B}$$

# Leading channels

$$\hat{\Delta}_{j_z=0}^{(-)} \sim \begin{pmatrix} k_x - ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix}$$

$\sim \text{He3-B}$

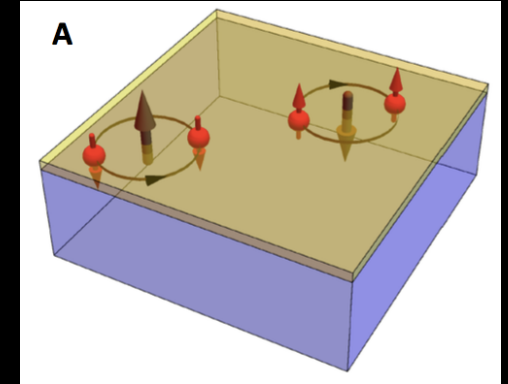


# Leading channels

$$\hat{\Delta}_{j_z=0}^{(-)} \sim \begin{pmatrix} k_x - ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix}$$

$\sim \text{He3-B}$

$$\hat{\Delta}_{j_z=\pm 1}^{(-)} \sim \begin{pmatrix} 0 & k_x \pm ik_y \\ k_x \pm ik_y & 0 \end{pmatrix}$$



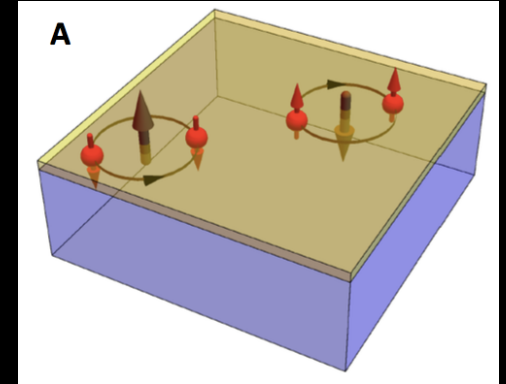
# Leading channels

$$\hat{\Delta}_{j_z=0}^{(-)} \sim \begin{pmatrix} k_x - ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix}$$

$\sim \text{He3-B}$

$$\hat{\Delta}_{j_z=\pm 1}^{(-)} \sim \begin{pmatrix} 0 & k_x \pm ik_y \\ k_x \pm ik_y & 0 \end{pmatrix}$$

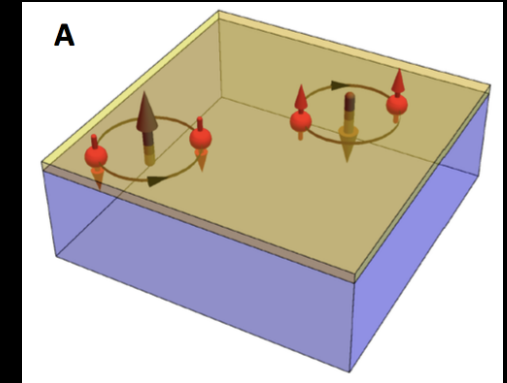
$\sim \text{He3-A}$



# Leading channels

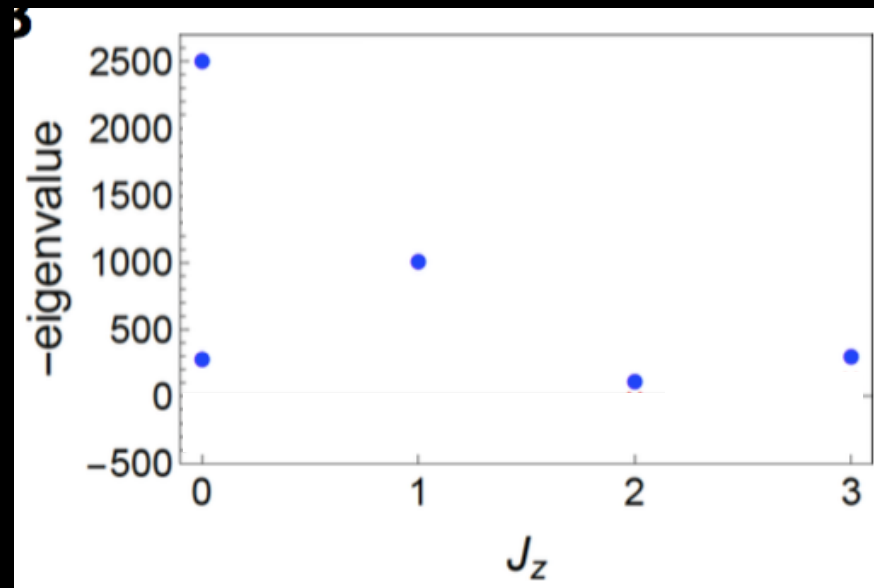
$$\hat{\Delta}_{j_z=0}^{(-)} \sim \begin{pmatrix} k_x - ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix}$$

$\sim \text{He3-B}$



$$\hat{\Delta}_{j_z=\pm 1}^{(-)} \sim \begin{pmatrix} 0 & k_x \pm ik_y \\ k_x \pm ik_y & 0 \end{pmatrix}$$

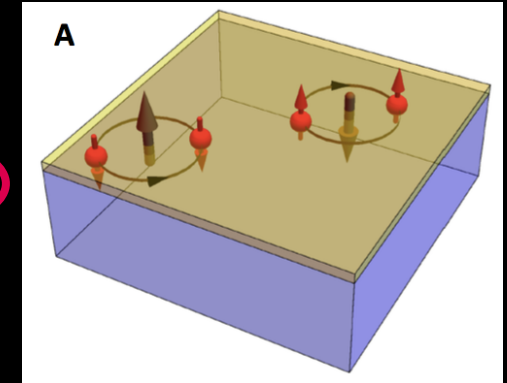
$\sim \text{He3-A}$



# Leading channels

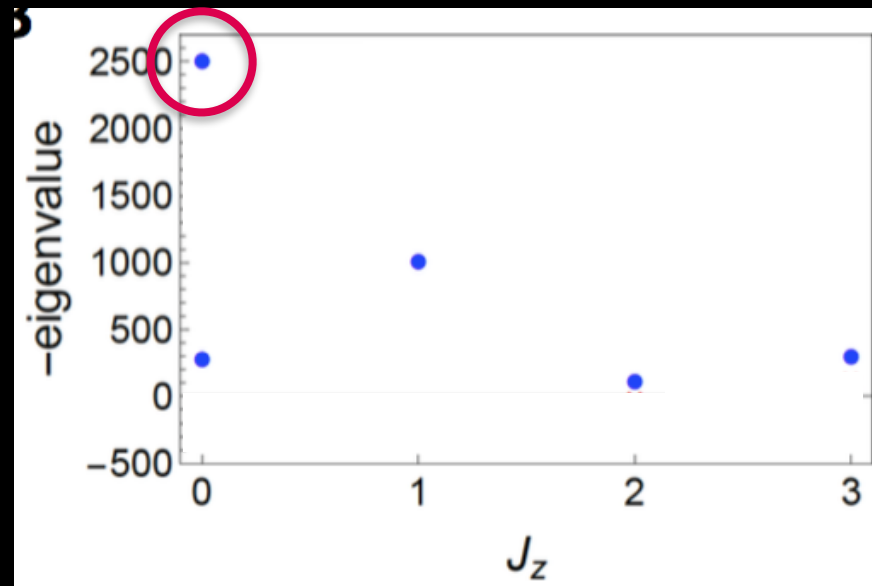
$$\hat{\Delta}_{j_z=0}^{(-)} \sim \begin{pmatrix} k_x - ik_y & 0 \\ 0 & k_x + ik_y \end{pmatrix}$$

$\sim \text{He3-B}$



$$\hat{\Delta}_{j_z=\pm 1}^{(-)} \sim \begin{pmatrix} 0 & k_x \pm ik_y \\ k_x \pm ik_y & 0 \end{pmatrix}$$

$\sim \text{He3-A}$





Can we persuade a material  
synthesis person?

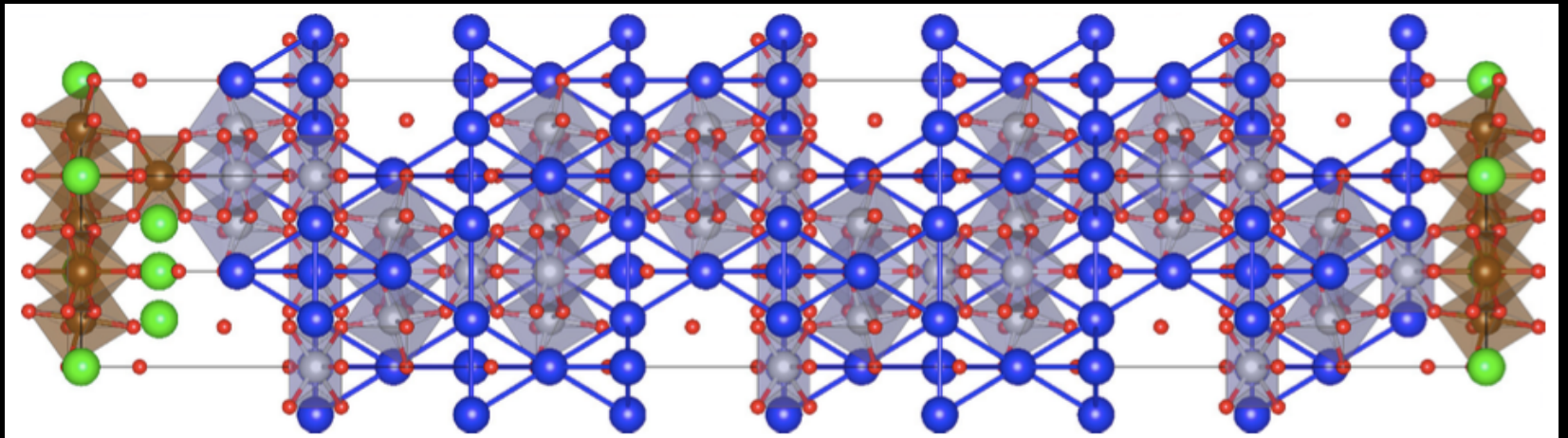
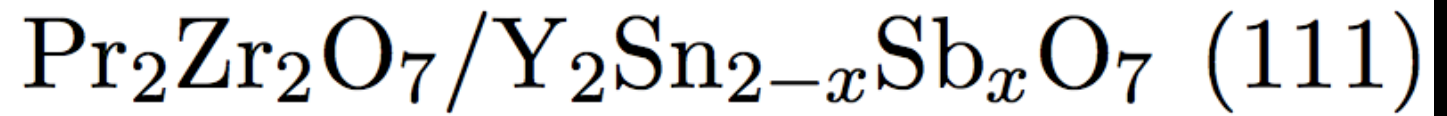
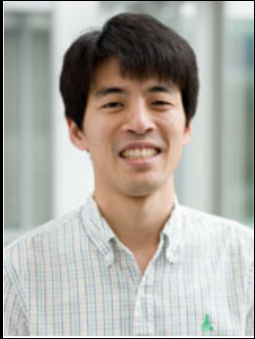
# Criteria for Metal

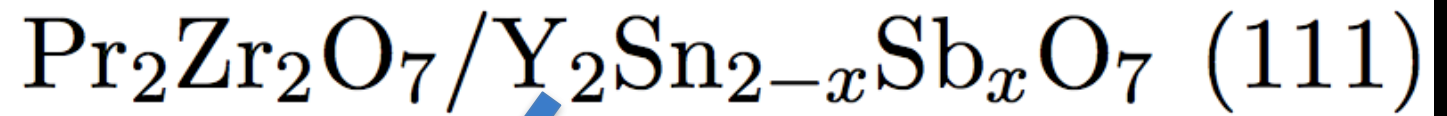
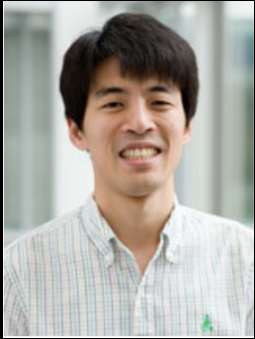
# Criteria for Metal

- Structural
  - ▶ Lattice match
    - ➡  $A_2B_2O_7$
  - ▶ No orphan bonds

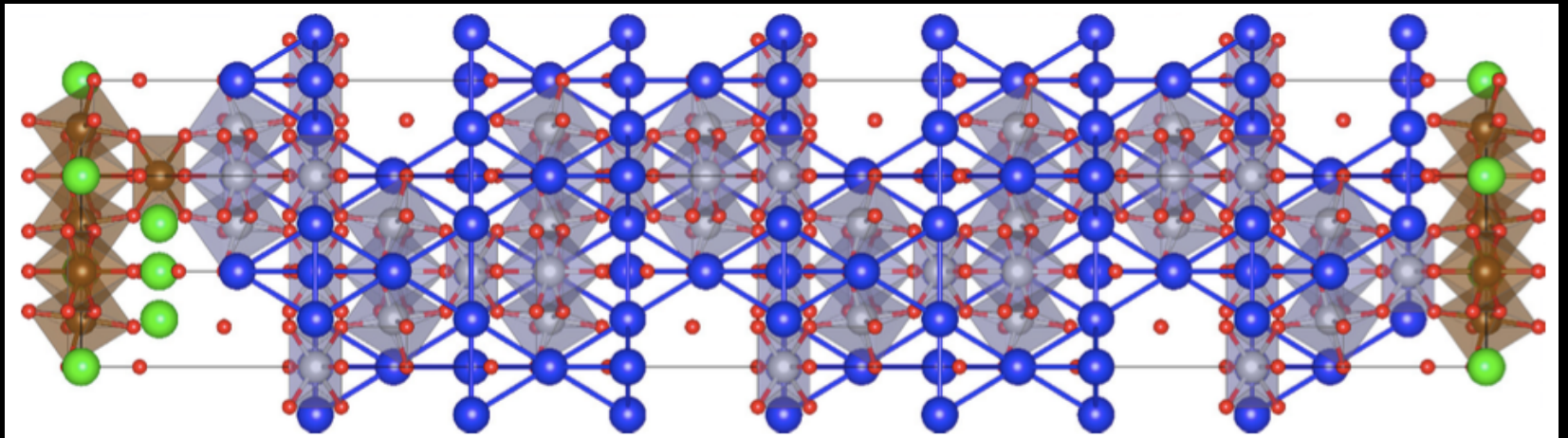
# Criteria for Metal

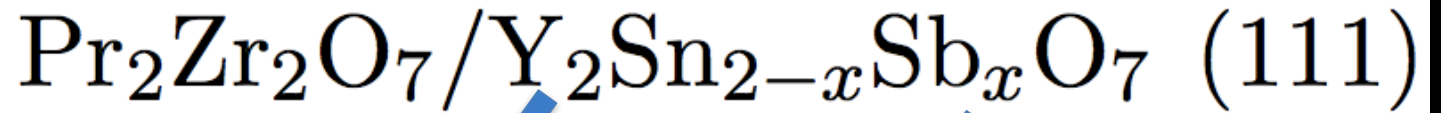
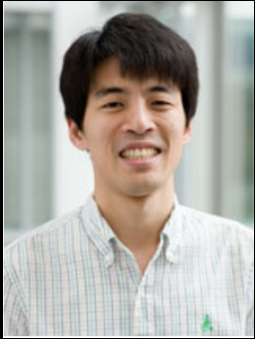
- Structural
  - ▶ Lattice match
    - ➡  $A_2B_2O_7$
  - ▶ No orphan bonds
- Electronic
  - ▶ Simple isotropic Fermi surface
  - ▶ Wave function penetration
  - ▶ Odd-# FS around high symmetry points





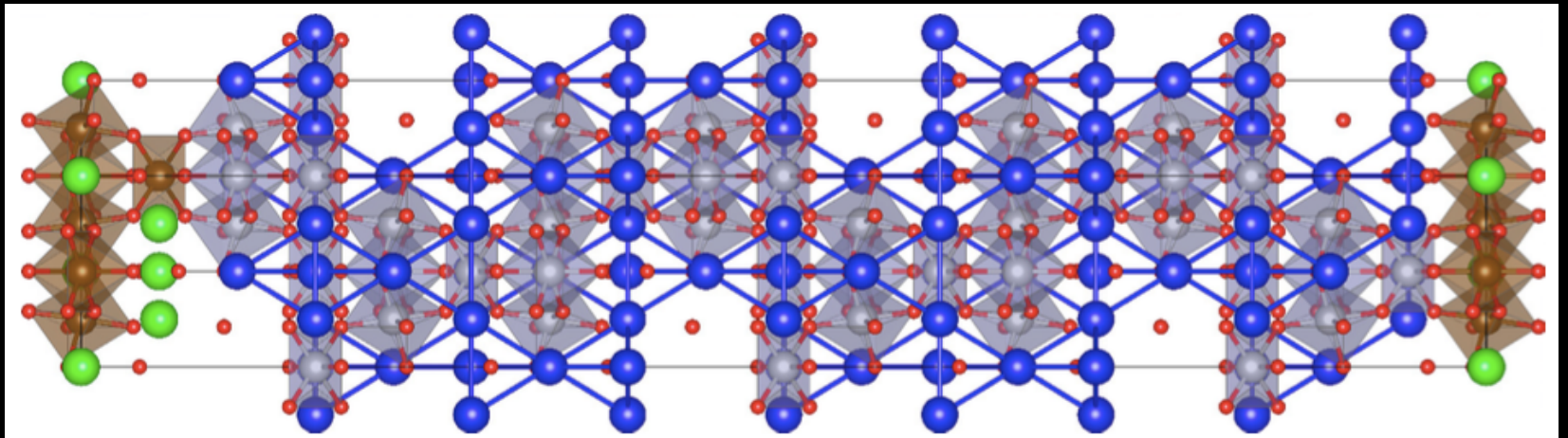
Non-magnetic



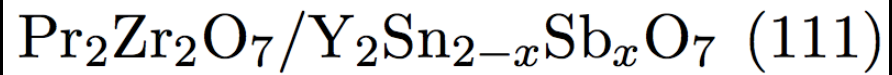


Non-magnetic

s-electrons:  
large overlap,  
isotropic FS.



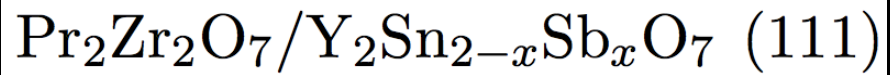
# Band structure for the Proposal



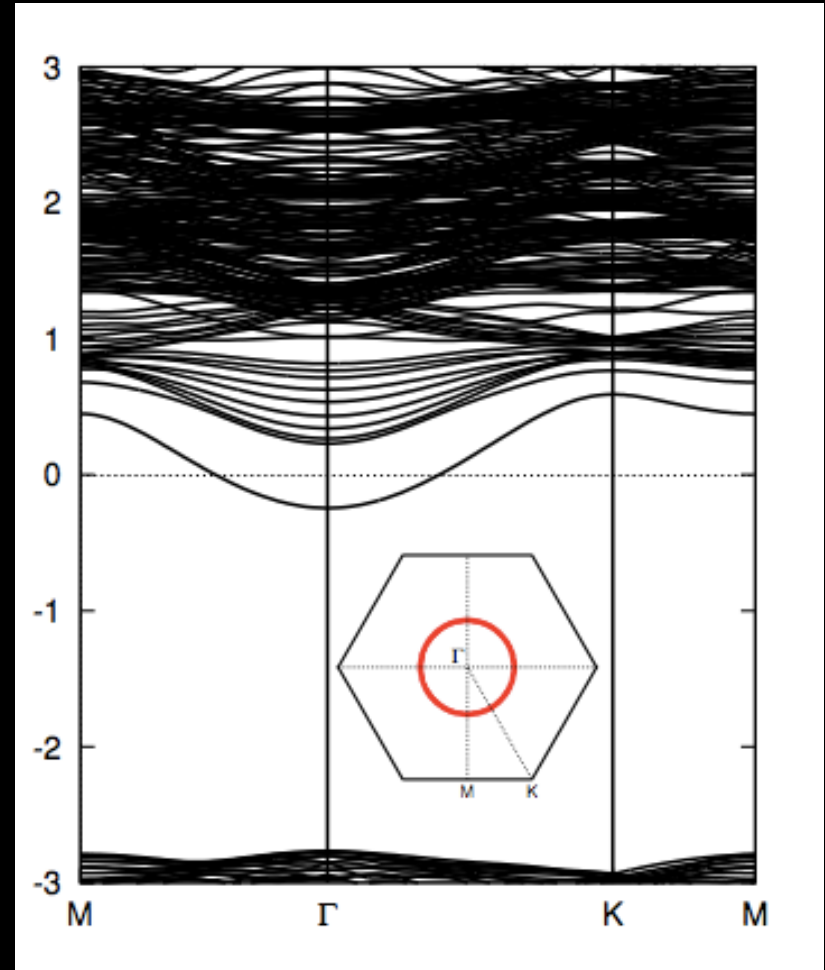
$$x=0.2$$



# Band structure for the Proposal



$$x=0.2$$

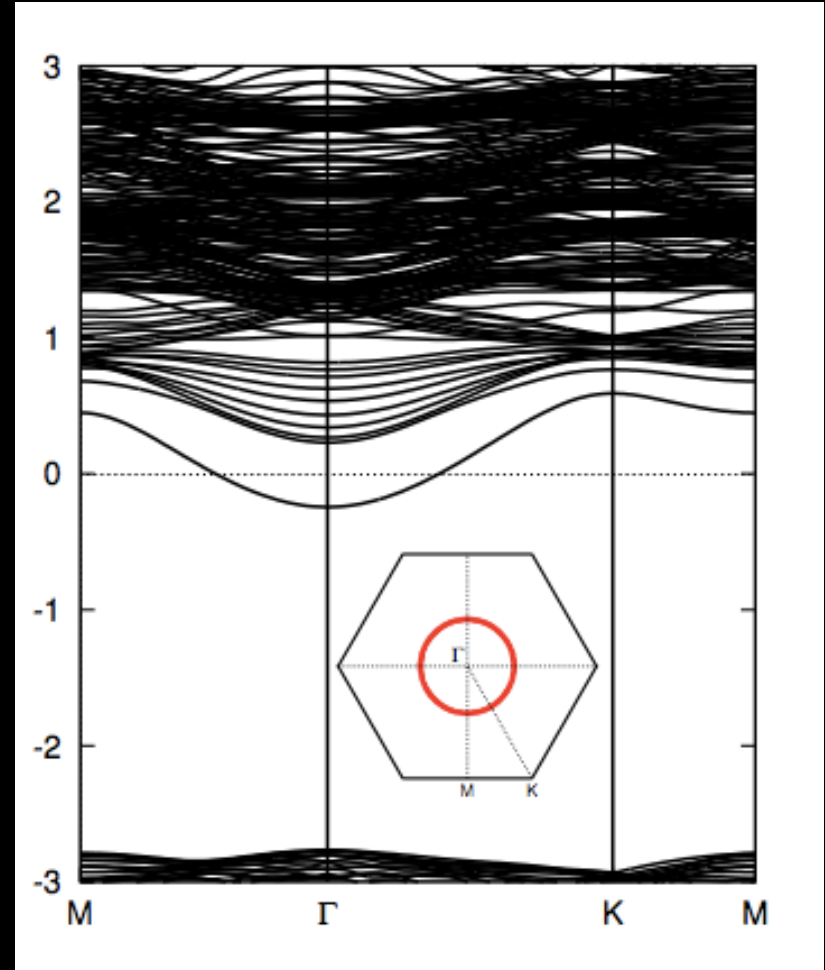


# Band structure for the Proposal

$\text{Pr}_2\text{Zr}_2\text{O}_7/\text{Y}_2\text{Sn}_{2-x}\text{Sb}_x\text{O}_7$  (111)

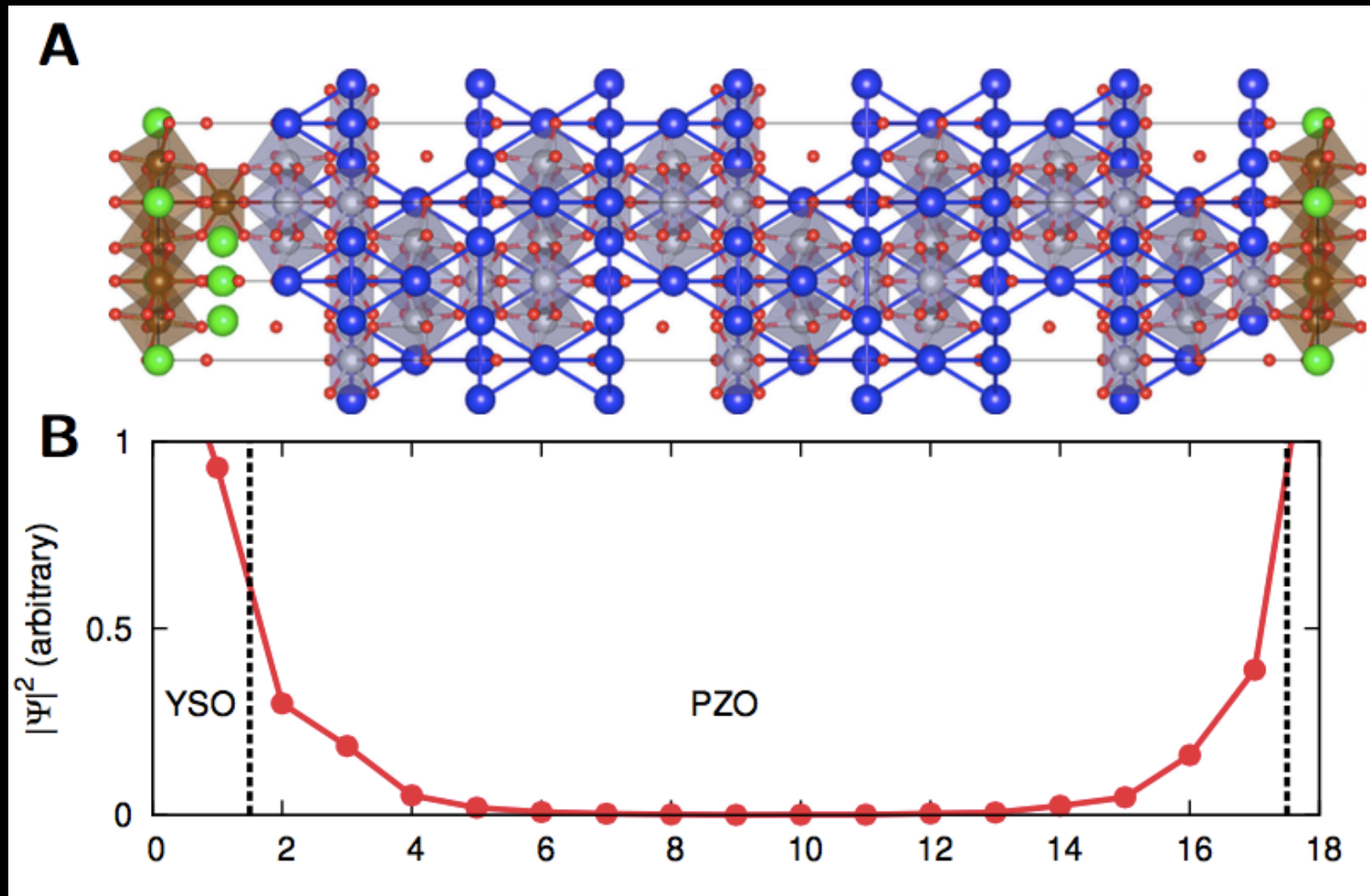
$x=0.2$

- Isotropic single pocket centered at  $\Gamma$ -point



# Wave function penetration

# Wave function penetration



# Earlier Proposal: Excitonic mechanism

# Earlier Proposal: Excitonic mechanism

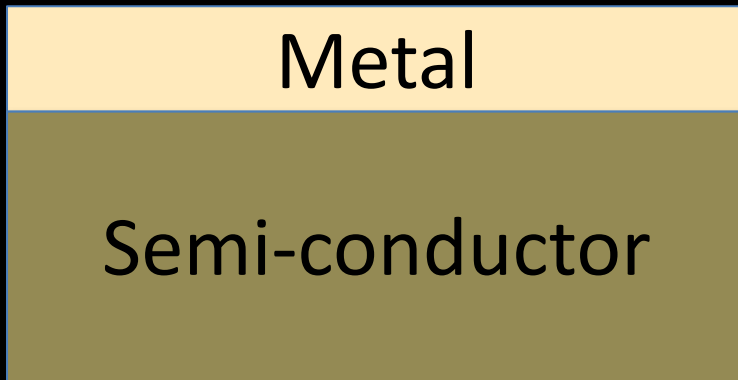


A diagram showing a vertical stack of two rectangular regions. The top region is light yellow and labeled 'Metal'. The bottom region is olive green and labeled 'Semi-conductor'. The two regions are separated by a thin horizontal line.

Metal

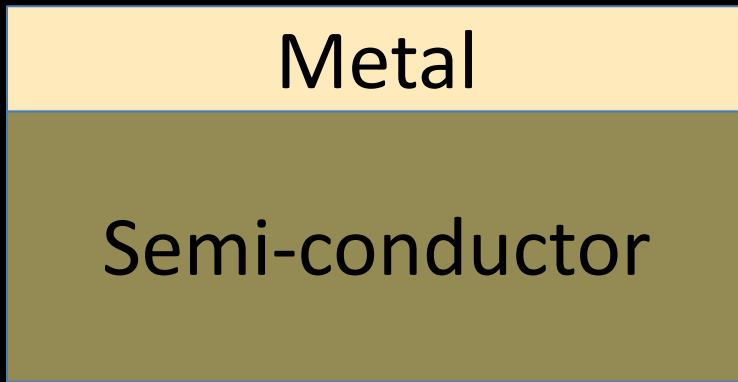
Semi-conductor

# Earlier Proposal: Excitonic mechanism



- Unstable against exchange.

# Earlier Proposal: Excitonic mechanism

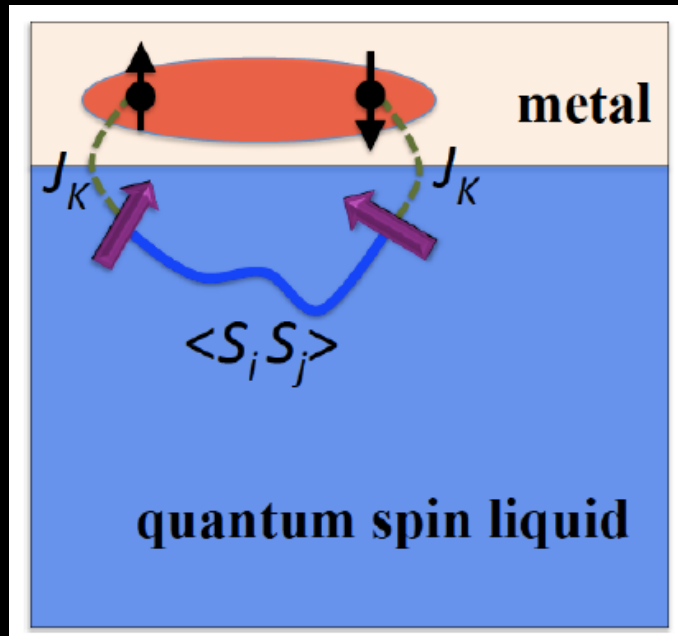


- Unstable against exchange.
- Intrinsically s-wave.

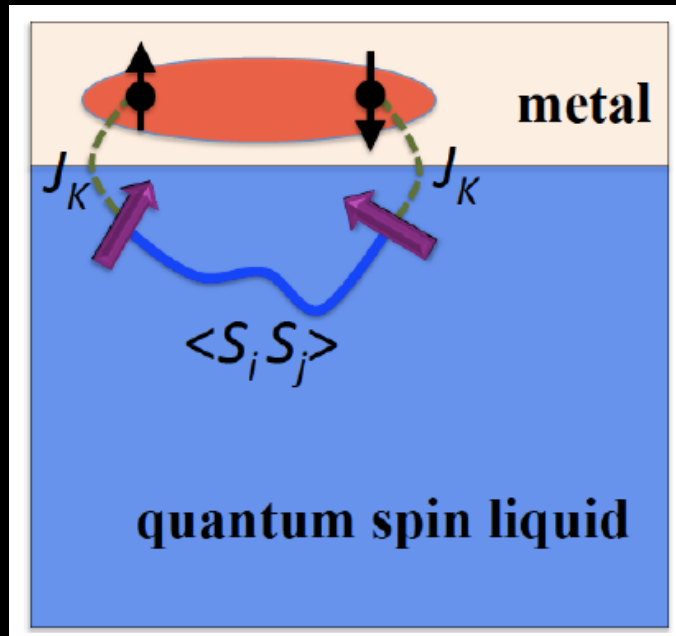
Little (64), Ginzburg (70), Bardeen (73)



# Topological Superconductivity in Metal/ Quantum-Spin-Ice Heterostructures

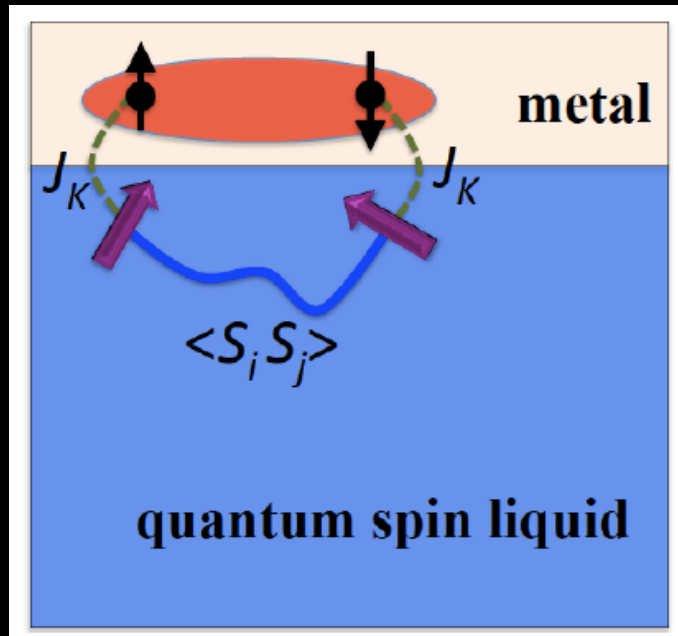


# Topological Superconductivity in Metal/ Quantum-Spin-Ice Heterostructures



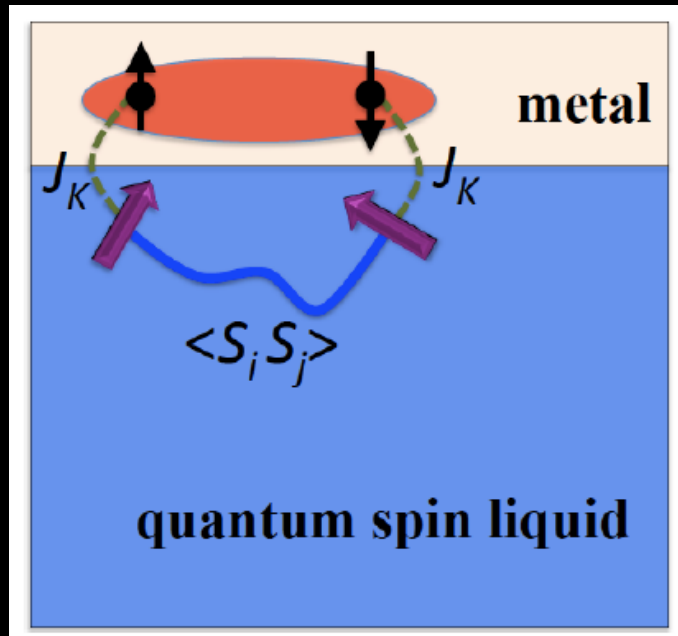
- Topological superconductor riding on QSL

# Topological Superconductivity in Metal/ Quantum-Spin-Ice Heterostructures



- Topological superconductor riding on QSL
- First T-inv Topo SC.

# Topological Superconductivity in Metal/ Quantum-Spin-Ice Heterostructures

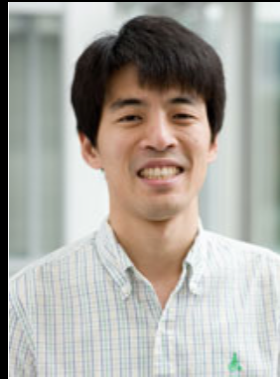


- Topological superconductor riding on QSL
- First T-inv Topo SC.
- Substantial phase space.

# Acknowledgements



Jian-huang She



Choonghyun Kim



Criag Fennie



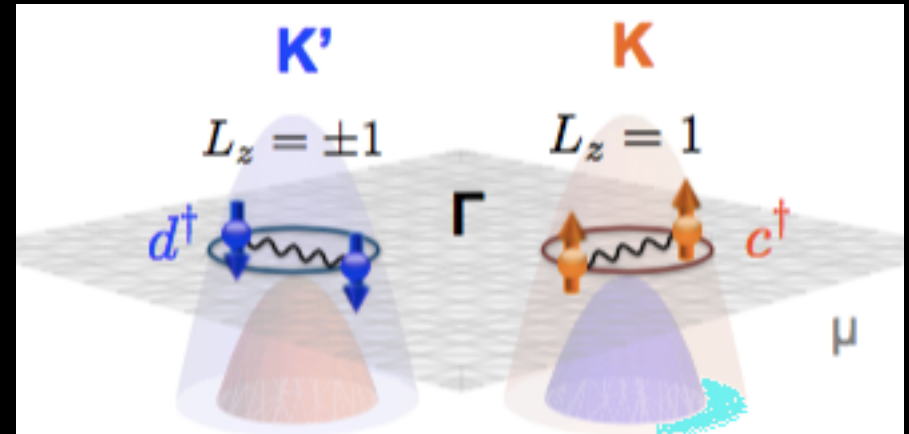
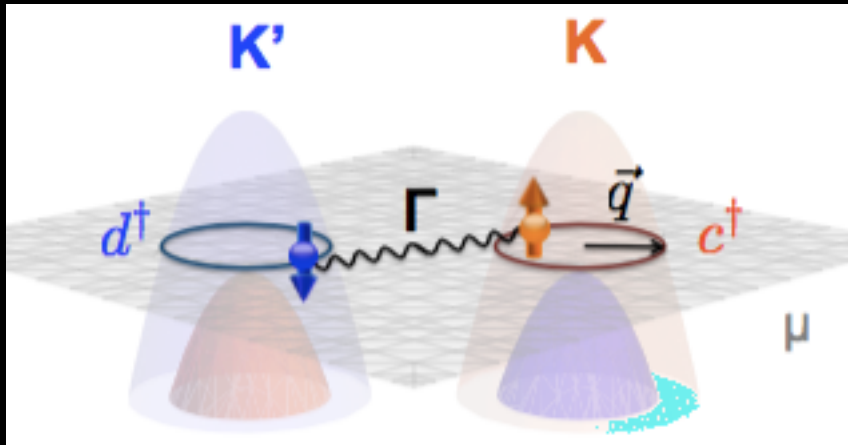
Michael Lawler

Funding: DOE, CCMR (NSF)

# Strategy II

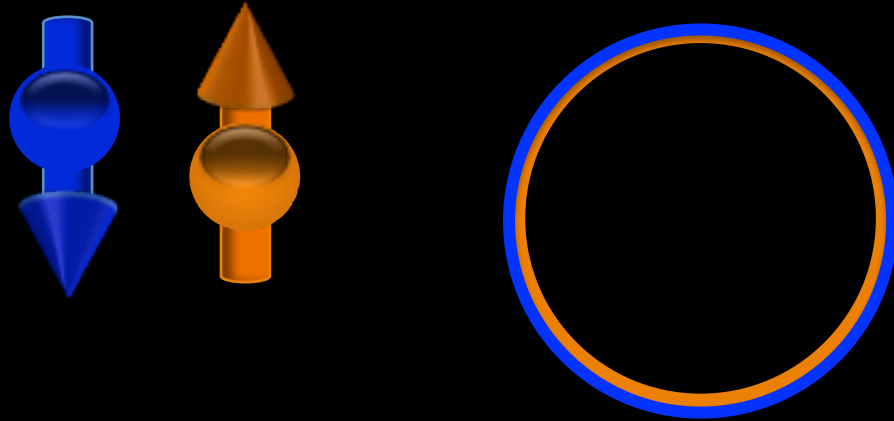
Manipulate the band  
structure

# Topological superconductivity in group-VI TMDs



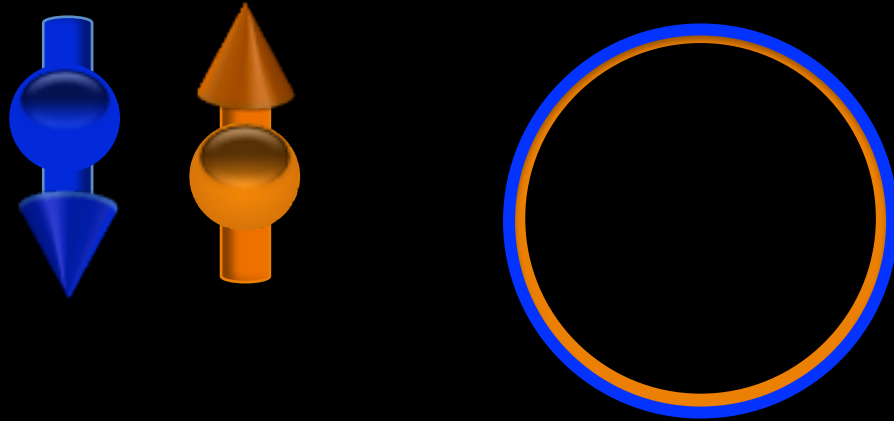
Yi-Ting Hsu, Abolhassan Vaezi, E-AK (in preparation)

# Spin-degenerate Fermi surface



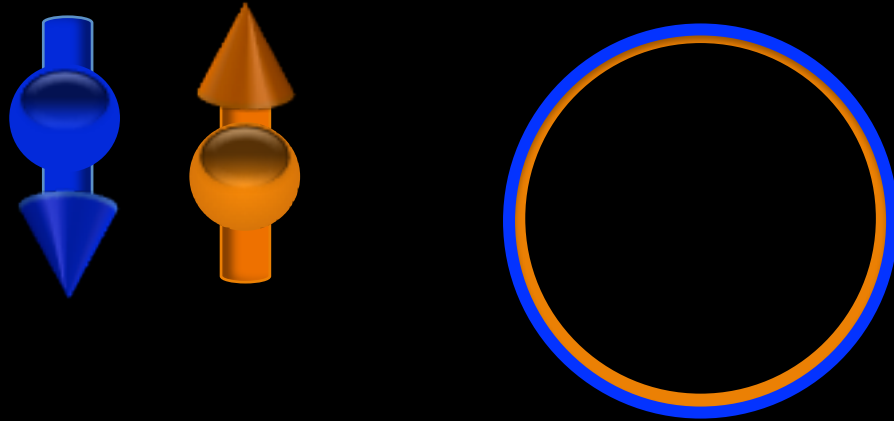


# Spin-degenerate Fermi surface



Singlet superconductor

# Spin-degenerate Fermi surface

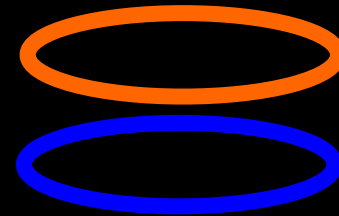


Singlet superconductor

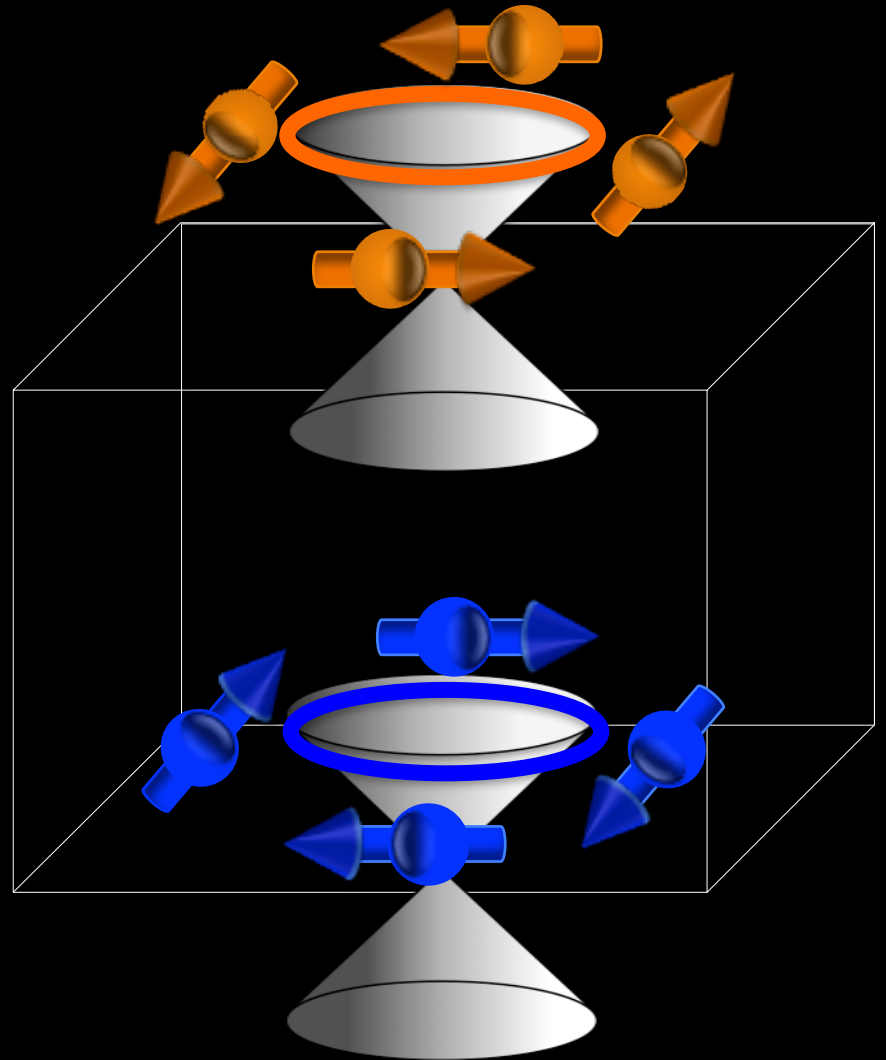
Q. What if the band structure is spin-split?

# Spinless fermion via **real space** splitting

# Spinless fermion via **real space** splitting

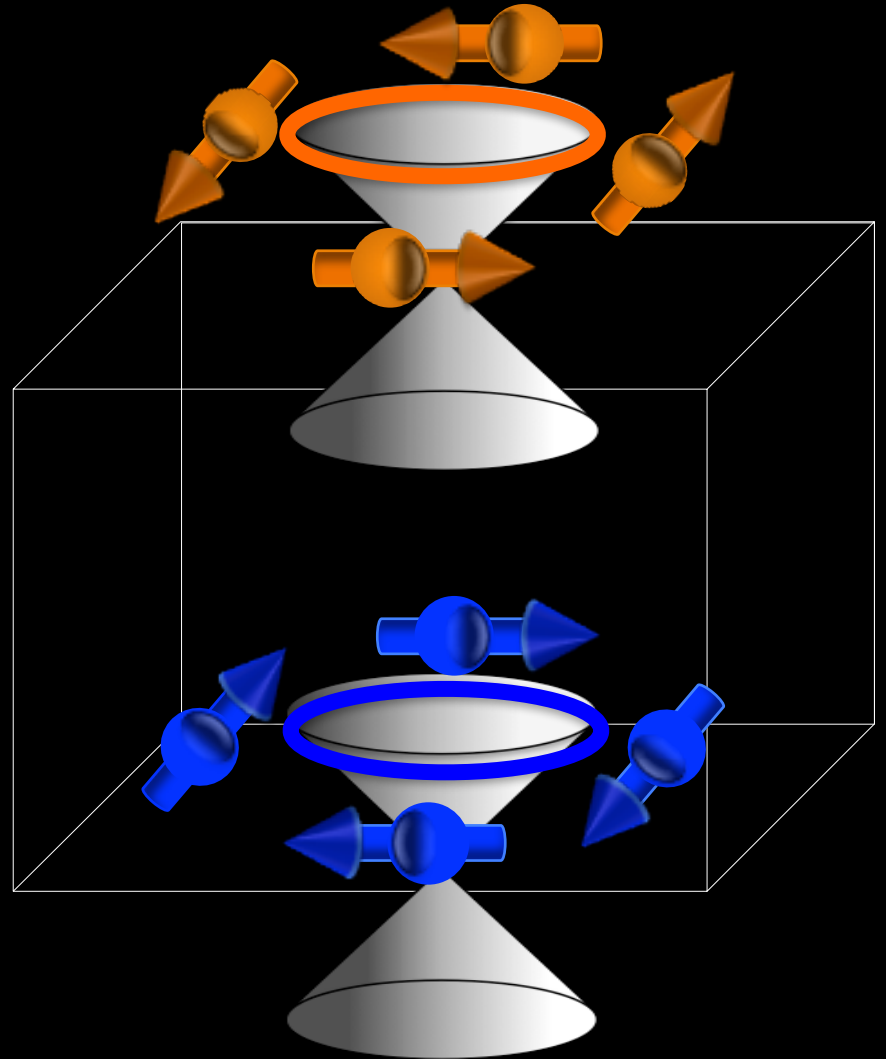


# Spinless fermion via **real space** splitting



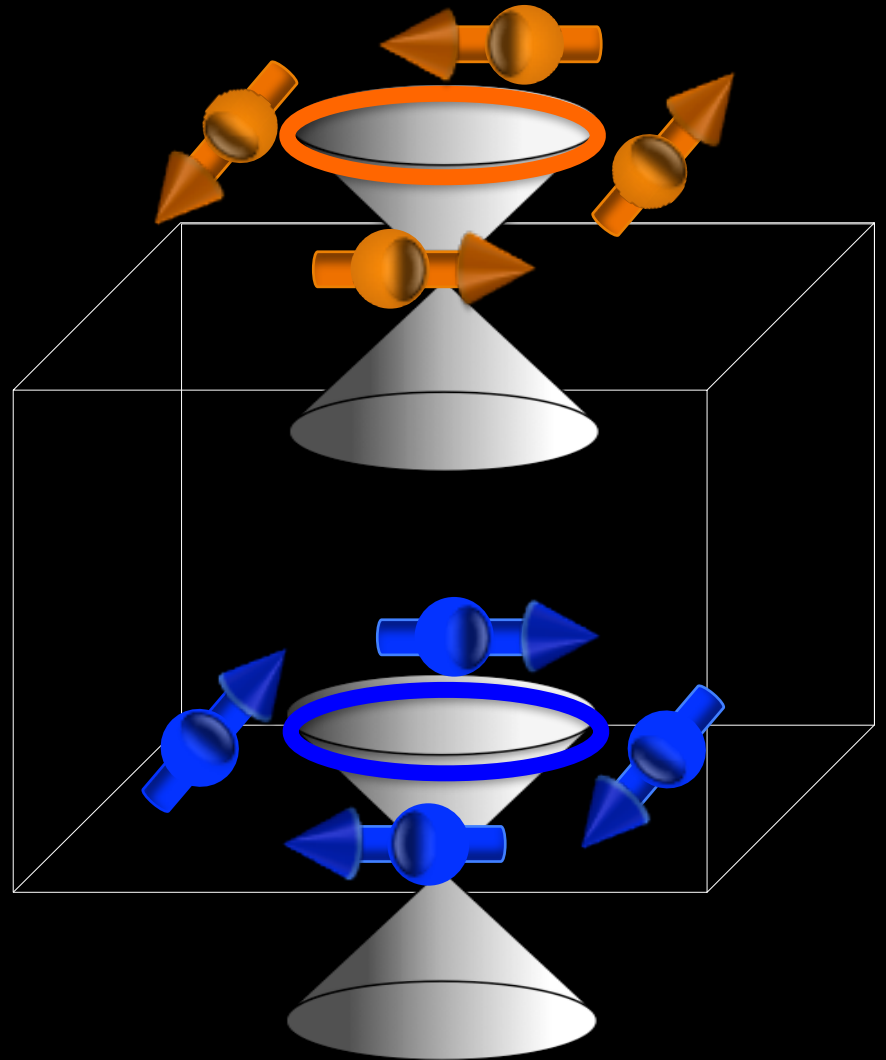
# Spinless fermion via **real space** splitting

- TI surface states



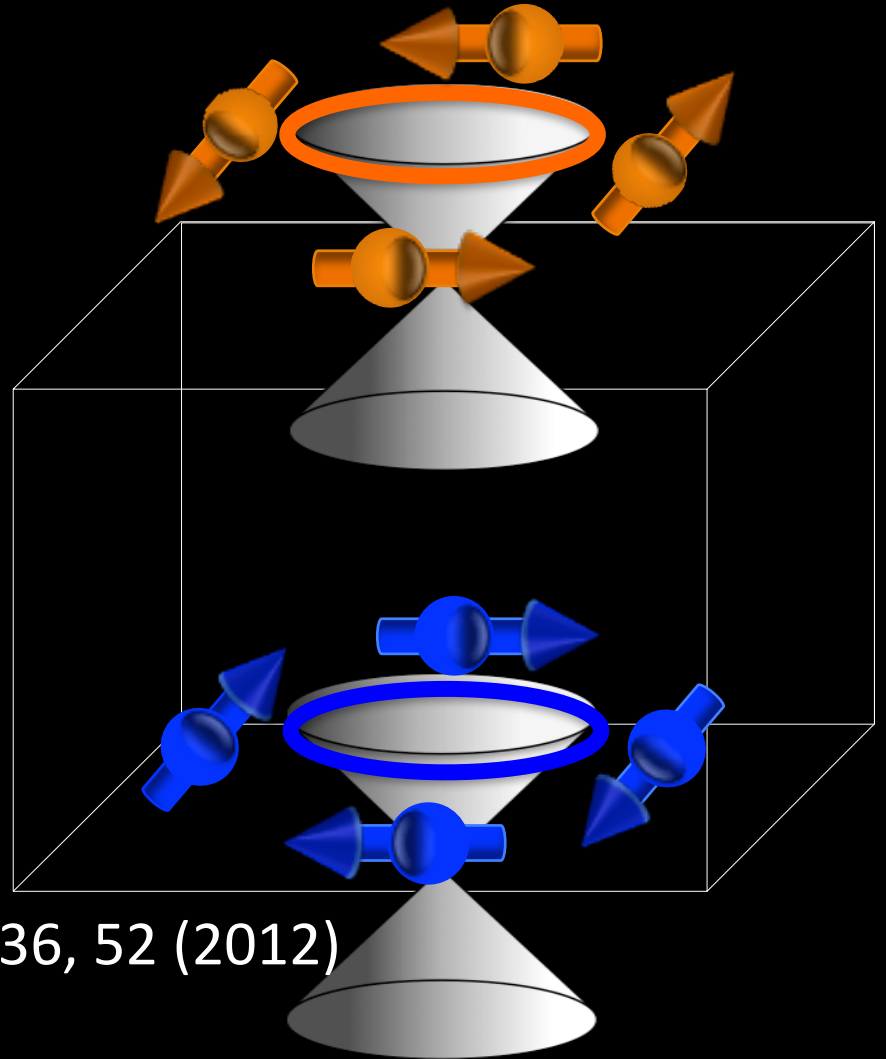
# Spinless fermion via **real space** splitting

- TI surface states
- Proximity induce topo SC



# Spinless fermion via **real space** splitting

- TI surface states
- Proximity induce topo SC



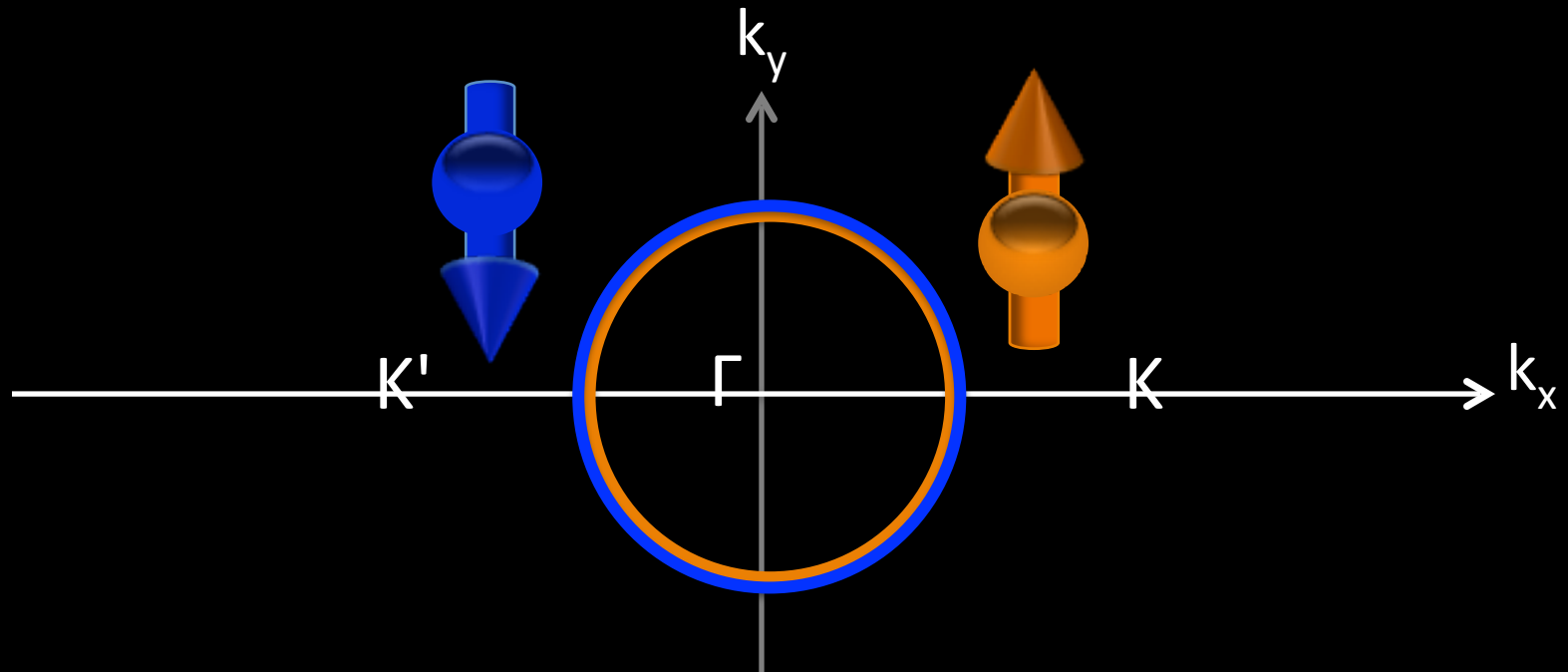
Fu & Kane, PRL (2008)

Experiments: Wang et al Science 336, 52 (2012)

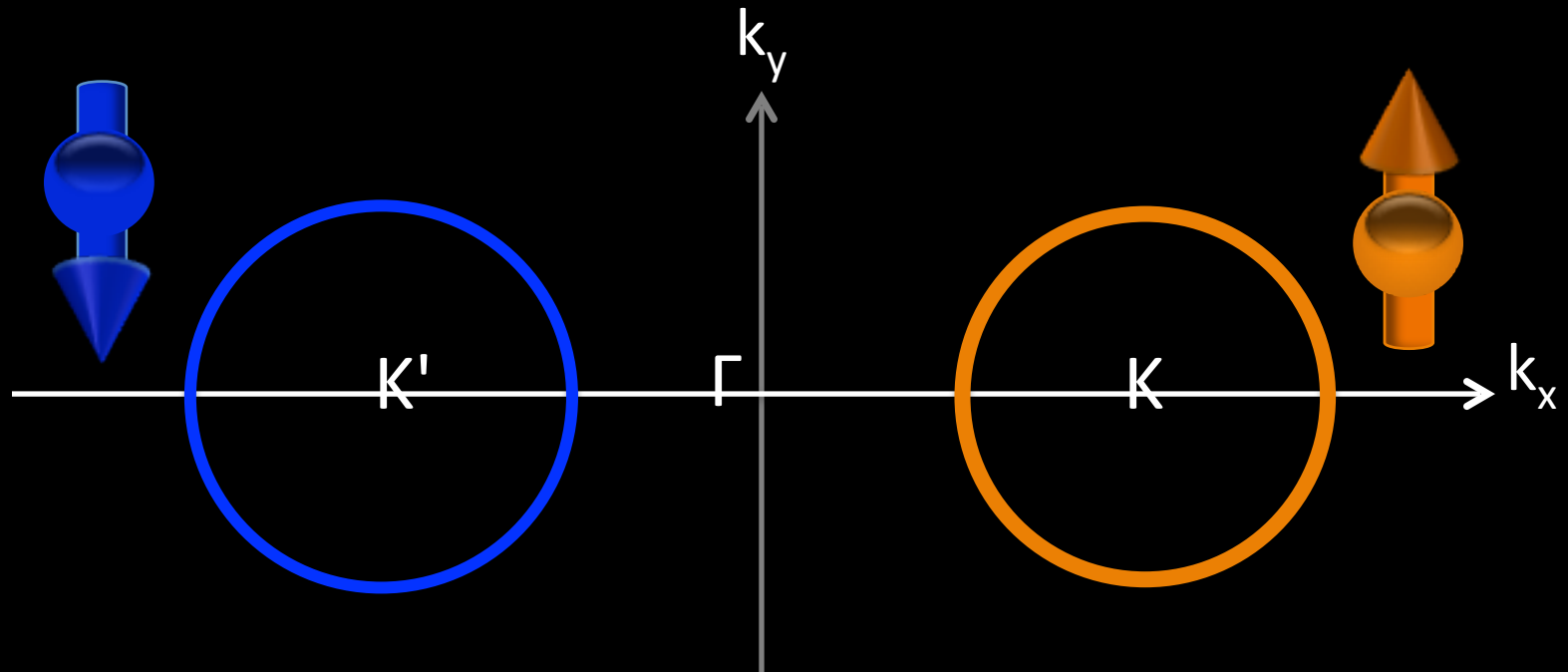
Xu et al, Nat.Phys 10, 943 (2014)



# Spinless fermion via **k-space** splitting?



# Spinless fermion via **k-space** splitting?



# Monolayer group VI TMD's

- **Non-centro symmetric**
  - ➔ Direct Gap  $\sim 2\text{eV}$
  - ➔ Dresselhaus spin-orbit

# Monolayer group VI TMD's

$\text{MoS}_2$ ,  $\text{WS}_2$ ,  $\text{MoSe}_2$ ,  $\text{WSe}_2$

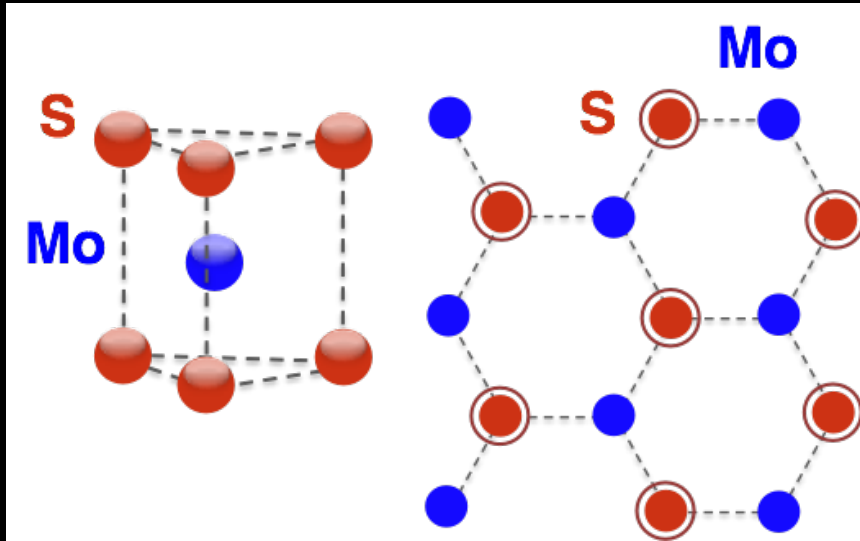
- **Non-centro symmetric**

- ➔ Direct Gap  $\sim 2\text{eV}$

- ➔ Dresselhaus spin-orbit

# Monolayer group VI TMD's

$\text{MoS}_2$ ,  $\text{WS}_2$ ,  $\text{MoSe}_2$ ,  $\text{WSe}_2$



- **Non-centro symmetric**

- ➔ Direct Gap  $\sim 2\text{eV}$

- ➔ Dresselhaus spin-orbit

# Band-selective spin-splitting

# Band-selective spin-splitting

- Partially filled crystal-field-split d-bands

# Band-selective spin-splitting

- Partially filled crystal-field-split d-bands
  - Conduction band  $|d_{z^2}\rangle: l_z=0$



# Band-selective spin-splitting

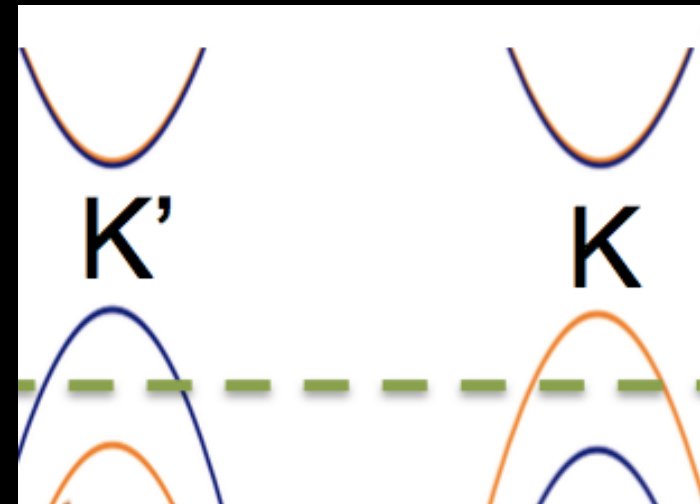
- Partially filled crystal-field-split d-bands
  - Conduction band  $|d_{z^2}\rangle : l_z=0$
  - Valence band  $\frac{1}{\sqrt{2}}(|d_{x^2-y^2}\rangle \mp i|d_{xy}\rangle) : l_z = \mp 1$

# Band-selective spin-splitting

- Partially filled crystal-field-split d-bands
  - Conduction band  $|d_{z^2}\rangle : l_z=0$
  - Valence band  $\frac{1}{\sqrt{2}}(|d_{x^2-y^2}\rangle \mp i|d_{xy}\rangle) : l_z = \mp 1$
- Spin-orbit coupling  $\vec{L} \cdot \vec{S}$

# Band-selective spin-splitting

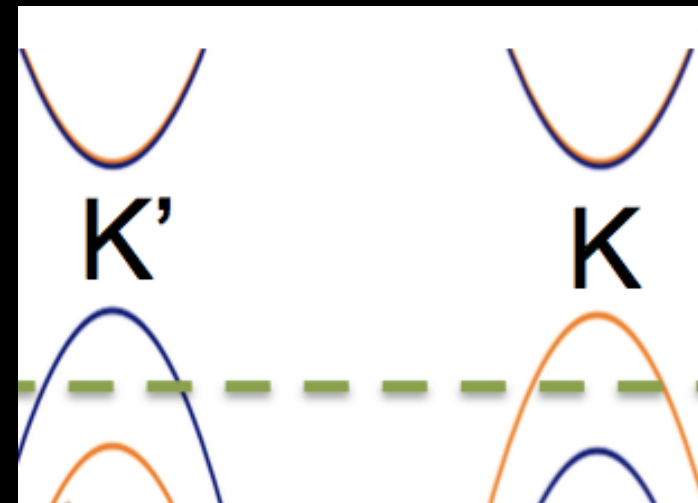
- Partially filled crystal-field-split d-bands
  - Conduction band  $|d_{z^2}\rangle : l_z=0$
  - Valence band  $\frac{1}{\sqrt{2}}(|d_{x^2-y^2}\rangle \mp i|d_{xy}\rangle) : l_z = \mp 1$
- Spin-orbit coupling  $\vec{L} \cdot \vec{S}$

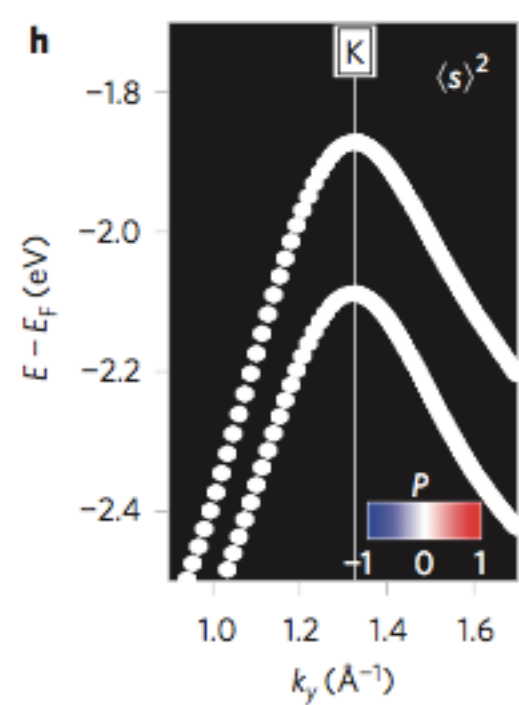
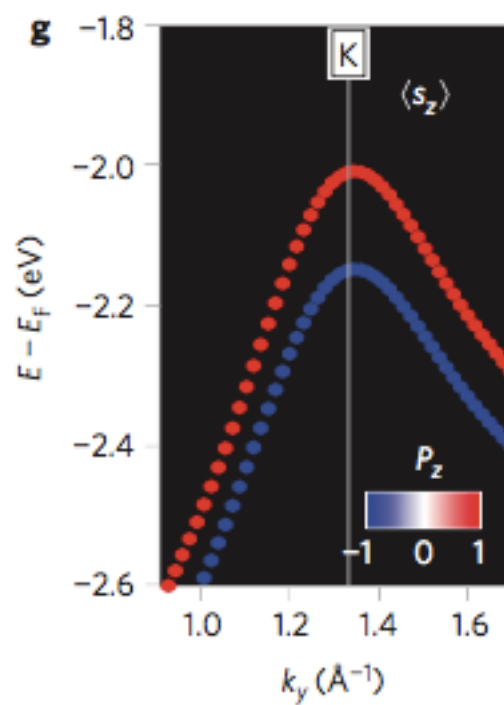
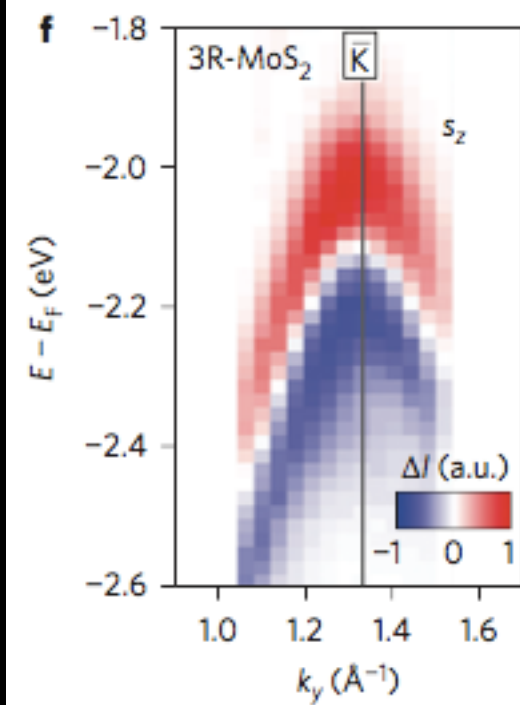


# Band-selective spin-splitting

- Partially filled crystal-field-split d-bands
  - Conduction band  $|d_{z^2}\rangle : l_z=0$
  - Valence band  $\frac{1}{\sqrt{2}}(|d_{x^2-y^2}\rangle \mp i|d_{xy}\rangle) : l_z = \mp 1$
- Spin-orbit coupling  $\vec{L} \cdot \vec{S}$

150~460meV



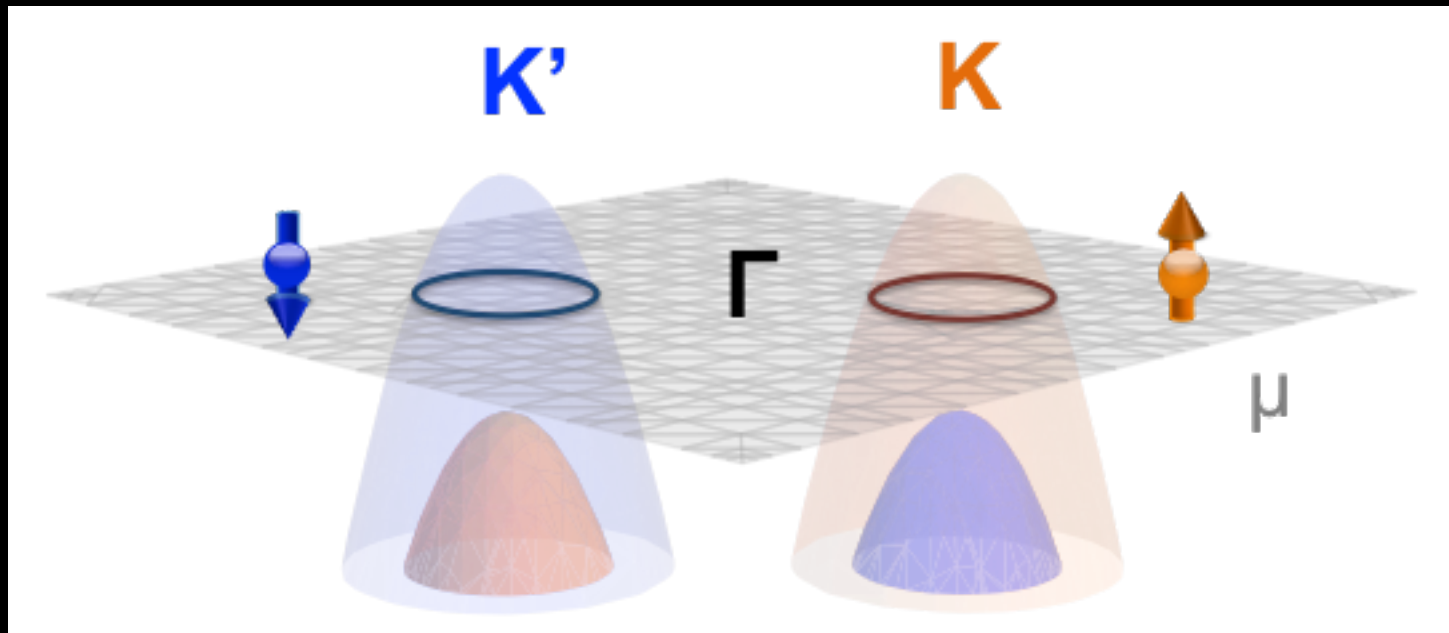


Iwasa group N. Nano (2014)

k-space spin-split FS?

# k-space spin-split FS?

## p-doped group VI- TMD!



Juice for superconductivity?



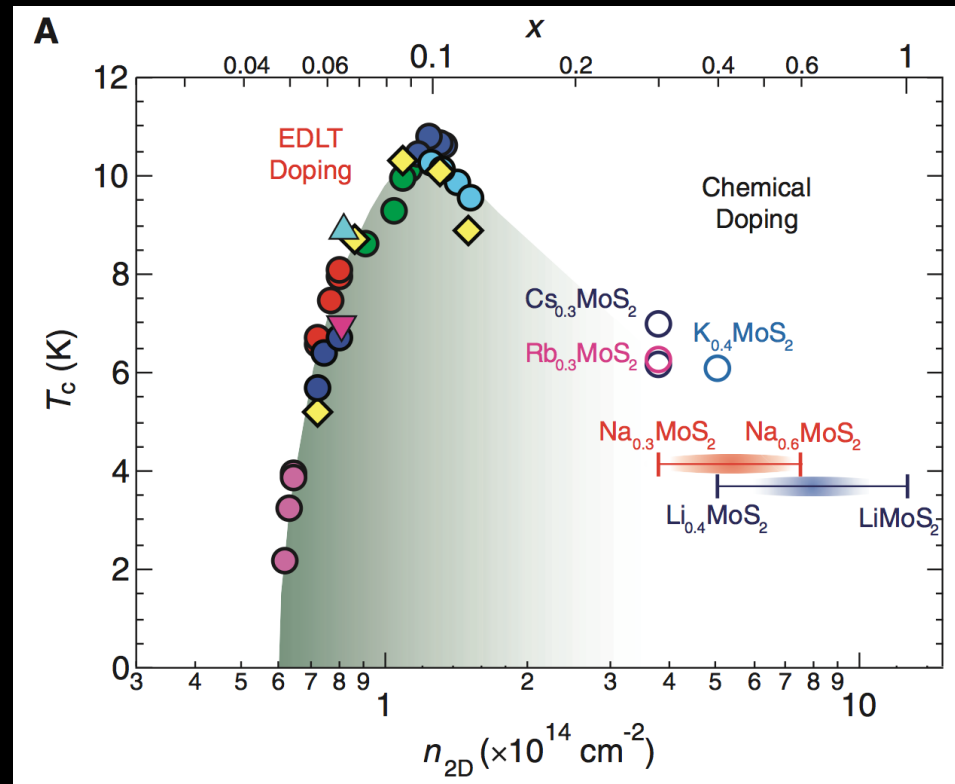
# Juice for superconductivity?

- d electrons => expect correlation effects

# Juice for superconductivity?

- d electrons => expect correlation effects

- n-doped TMD's  
*J.T.Ye et al. (Science 2012)*



p-doped TMD

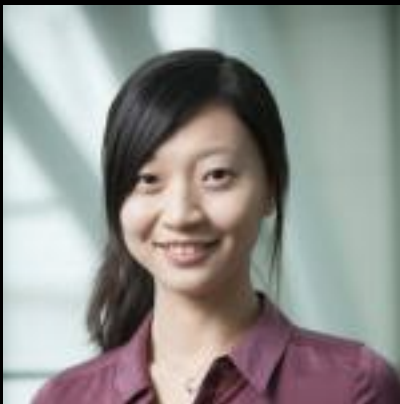
k-space spin-split Fermi surfaces  
+

Moderate correlation (d-electron)

p-doped TMD  
k-space spin-split Fermi surfaces  
+  
Moderate correlation (d-electron)



Topological SC?



Yi-Ting Hsu



Mark Fischer



Abolhassan Vaezi

# Model

- Kinetic term


$$H_0(\vec{q}) = at(\tau q_x \hat{\sigma}_x + q_y \hat{\sigma}_y) + \frac{\Delta}{2} \hat{\sigma}_z - \lambda \tau \hat{s}_z \otimes \frac{\hat{\sigma}_z - 1}{2}$$

- Repulsive interaction term

$$H'(W) = \sum_i U n_{i,\uparrow} n_{i,\downarrow}$$

# Model

- Kinetic term

$$H_0(\vec{q}) = at(\tau q_x \hat{\sigma}_x + q_y \hat{\sigma}_y) + \frac{\Delta}{2} \hat{\sigma}_z - \lambda \tau \hat{s}_z \otimes \frac{\hat{\sigma}_z - 1}{2}$$


- Repulsive interaction term

$$H'(W) = \sum_i U n_{i,\uparrow} n_{i,\downarrow}$$

# Model

- Kinetic term

$$H_0(\vec{q}) = at(\tau q_x \hat{\sigma}_x + q_y \hat{\sigma}_y) + \frac{\Delta}{2} \hat{\sigma}_z - \lambda \tau \hat{s}_z \otimes \frac{\hat{\sigma}_z - 1}{2}$$


Band-basis

- Repulsive interaction term

$$H'(W) = \sum_i U n_{i,\uparrow} n_{i,\downarrow}$$

# Model

- Kinetic term

$$H_0(\vec{q}) = at(\tau q_x \hat{\sigma}_x + q_y \hat{\sigma}_y) + \frac{\Delta}{2} \hat{\sigma}_z - \lambda \tau \hat{s}_z \otimes \frac{\hat{\sigma}_z - 1}{2}$$


Band-basis

- Repulsive interaction term

$$H'(W) = \sum_i U n_{i,\uparrow} n_{i,\downarrow}$$



# Model

- Kinetic term

$$H_0(\vec{q}) = at(\tau q_x \hat{\sigma}_x + q_y \hat{\sigma}_y) + \frac{\Delta}{2} \hat{\sigma}_z - \lambda \tau \hat{s}_z \otimes \frac{\hat{\sigma}_z - 1}{2}$$

Band-basis

Spin-basis

- Repulsive interaction term

$$H'(W) = \sum_i U n_{i,\uparrow} n_{i,\downarrow}$$

# Superconductivity out of repulsive interaction?

(Kohn & Luttinger 1965)

# Superconductivity out of repulsive interaction?

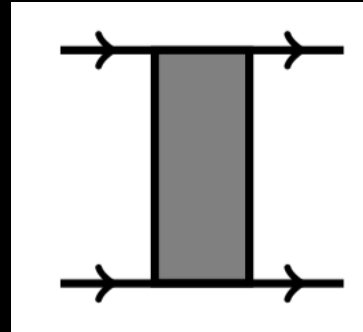
- Kohn-Luttinger: singularity in scattering amplitude  $\Gamma(\vec{q})$

(Kohn & Luttinger 1965)

# Superconductivity out of repulsive interaction?

- Kohn-Luttinger: singularity in scattering amplitude  $\Gamma(\vec{q})$

(Kohn & Luttinger 1965)

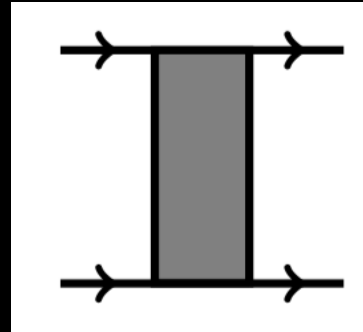


# Superconductivity out of repulsive interaction?

- Kohn-Luttinger: singularity in scattering amplitude  $\Gamma(\vec{q})$

➡ Non-s wave

(Kohn & Luttinger 1965)

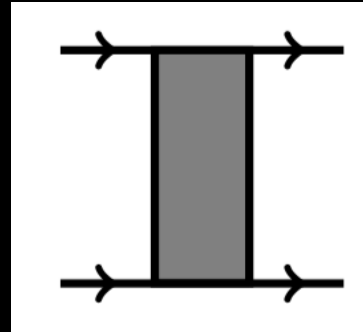


# Superconductivity out of repulsive interaction?

- Kohn-Luttinger: singularity in scattering amplitude  $\Gamma(\vec{q})$

➡ Non-s wave

(Kohn & Luttinger 1965)



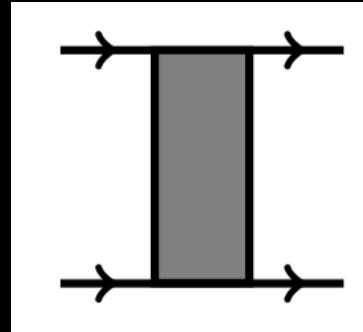
- Two-step RG formulation  
: Fe-based SC, doped graphene, SrRuO

# Superconductivity out of repulsive interaction?

- Kohn-Luttinger: singularity in scattering amplitude  $\Gamma(\vec{q})$

➔ Non-s wave

(Kohn & Luttinger 1965)



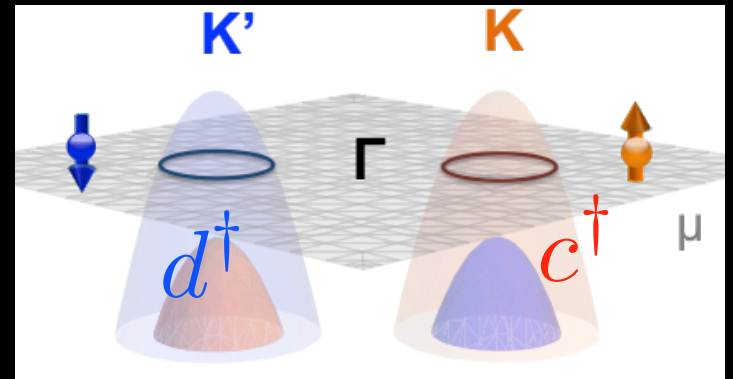
- Two-step RG formulation  
: Fe-based SC, doped graphene, SrRuO

Chubukov & Nandkishore, Raghu & Kivelson (2008 - 2012)

Two-step RG on p-doped TMD

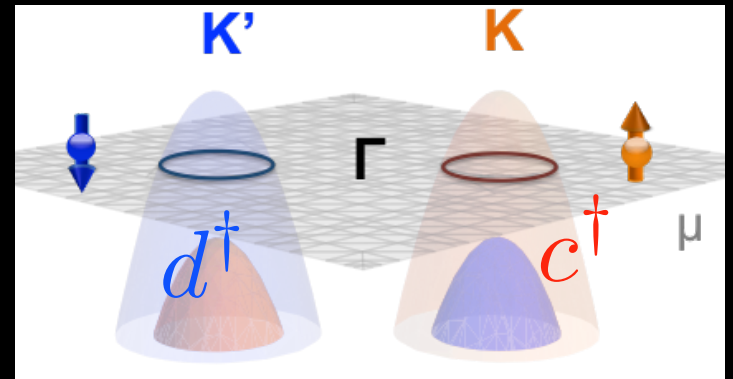


Step I:  $W \rightarrow \Lambda_0$



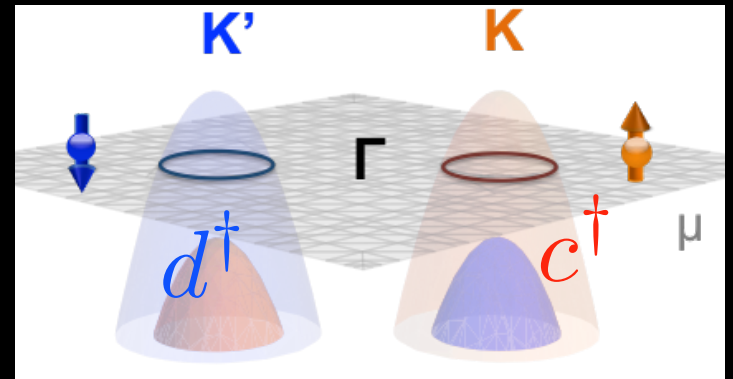
- At scale  $\Lambda_0$ : Effective model

Step I:  $W \rightarrow \Lambda_0$



- At scale  $W$ : Microscopic model
- At scale  $\Lambda_0$ : Effective model

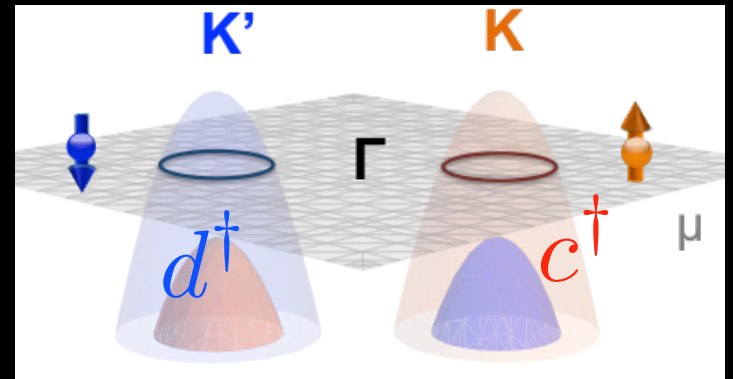
# Step I: $W \rightarrow \Lambda_0$



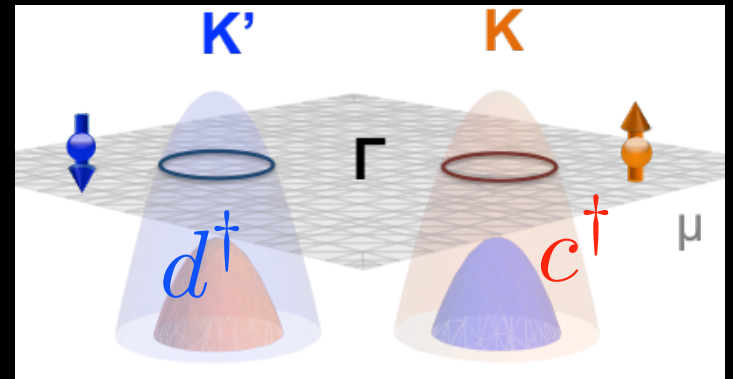
- At scale  $W$ : Microscopic model
- At scale  $\Lambda_0$ : Effective model

$$H'_{eff}(\Lambda_0) = \sum_{\vec{q}, \vec{q}'} g_{\text{inter}}^{(0)}(\vec{q}, \vec{q}') c_{\vec{q}'}^\dagger d_{-\vec{q}'}^\dagger d_{-\vec{q}} c_{\vec{q}} \\ + g_{\text{intra}}^{(0)}(\vec{q}, \vec{q}') d_{\vec{q}'}^\dagger d_{-\vec{q}'}^\dagger d_{-\vec{q}} d_{\vec{q}} + (c \leftrightarrow d)$$

Step I:  $W \rightarrow \Lambda_0$

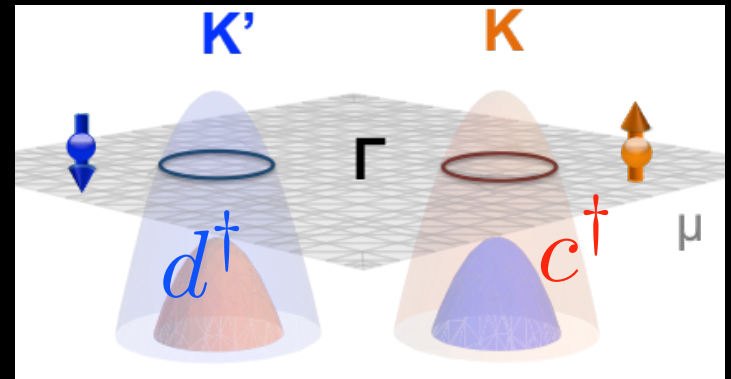


Step I:  $W \rightarrow \Lambda_0$

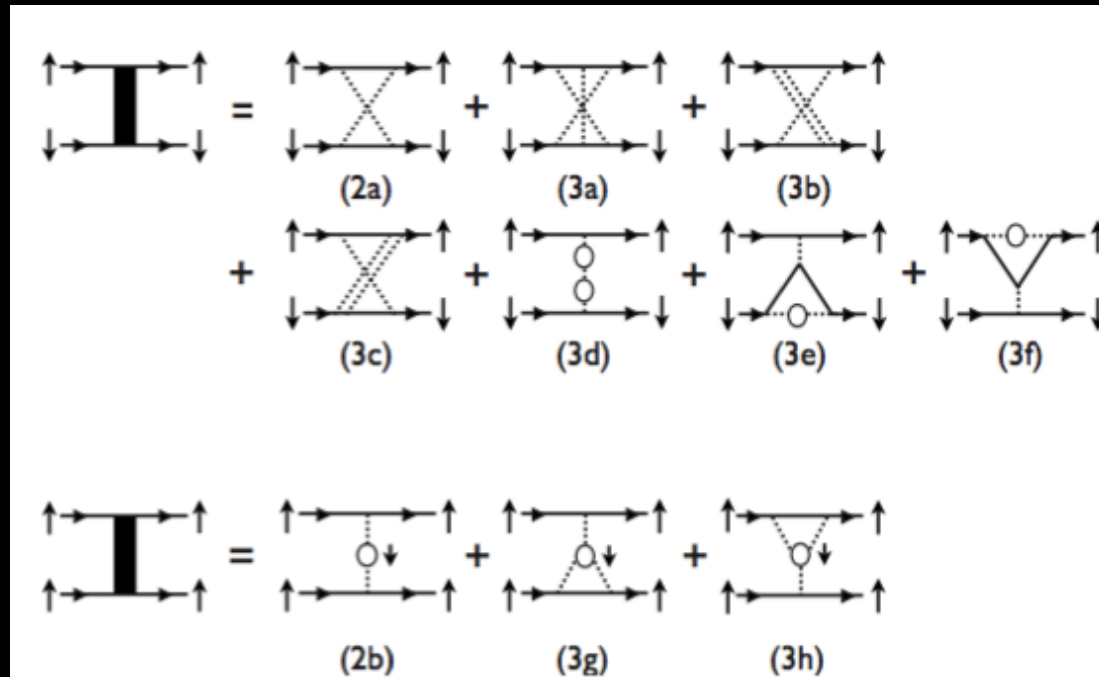


- $g_{\text{intra},0}$  and  $g_{\text{inter},0}$  at two-loop

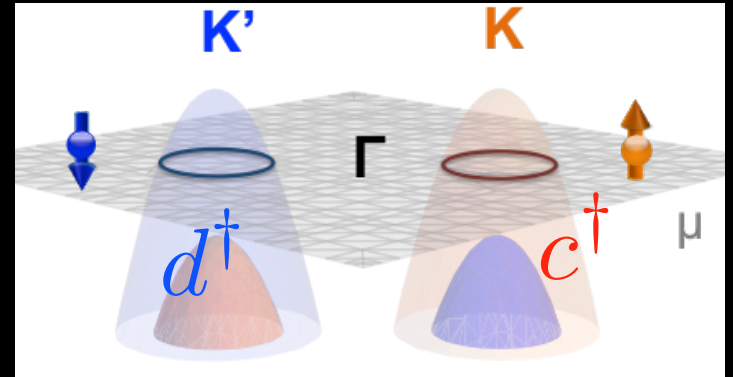
# Step I: $W \rightarrow \Lambda_0$



- $g_{\text{intra},0}$  and  $g_{\text{inter},0}$  at two-loop



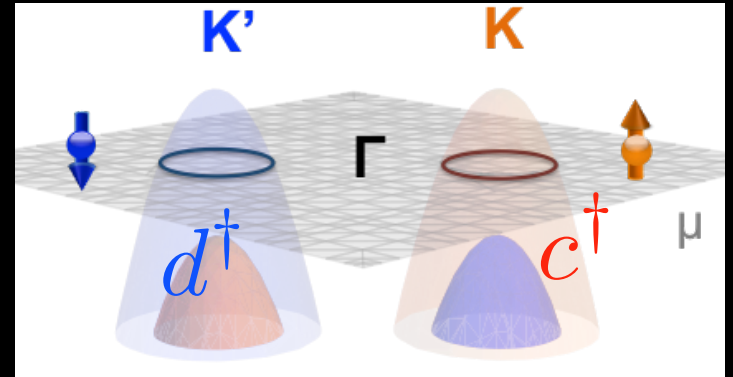
Step I:  $W \rightarrow \Lambda_0$



- $g_{\text{intra},0}$  and  $g_{\text{inter},0}$  at two-loop

$$g_{\text{inter}}^{(0)}(\vec{q}, \vec{q}') = U + U^3 f_{\text{inter}}(\vec{q}, \vec{q}')$$

Step I:  $W \rightarrow \Lambda_0$



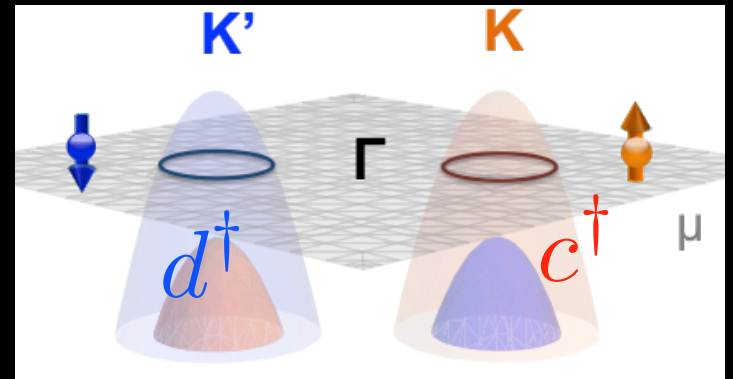
- $g_{\text{intra},0}$  and  $g_{\text{inter},0}$  at two-loop

$$g_{\text{inter}}^{(0)}(\vec{q}, \vec{q}') = U + U^3 f_{\text{inter}}(\vec{q}, \vec{q}')$$

$$g_{\text{intra}}^{(0)}(\vec{q}, \vec{q}') = U^3 f_{\text{intra}}(\vec{q}, \vec{q}')$$



Step I:  $W \rightarrow \Lambda_0$



- $g_{\text{intra},0}$  and  $g_{\text{inter},0}$  at two-loop

$$g_{\text{inter}}^{(0)}(\vec{q}, \vec{q}') = U + U^3 f_{\text{inter}}(\vec{q}, \vec{q}')$$

$$g_{\text{intra}}^{(0)}(\vec{q}, \vec{q}') = U^3 f_{\text{intra}}(\vec{q}, \vec{q}')$$

- $f$ 's  $< 0 \rightarrow g^{(0)}$ 's  $< 0$  in anisotropic channel



Step I:  $\Lambda_0 \rightarrow 0$

- RG flow
- Divergence if  $\lambda^{(0)} < 0$

## Step I: $\Lambda_0 \rightarrow 0$

- RG flow

$$\frac{d\lambda}{dy} = -\lambda^2$$

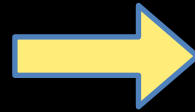
$$y \equiv \nu_0 \text{Log}(\Lambda_0/E)$$

- Divergence if  $\lambda^{(0)} < 0$

## Step I: $\Lambda_0 \rightarrow 0$

- RG flow

$$\frac{d\lambda}{dy} = -\lambda^2$$



$$\lambda(y) = \frac{\lambda^{(0)}}{1 + \lambda^{(0)}y}$$

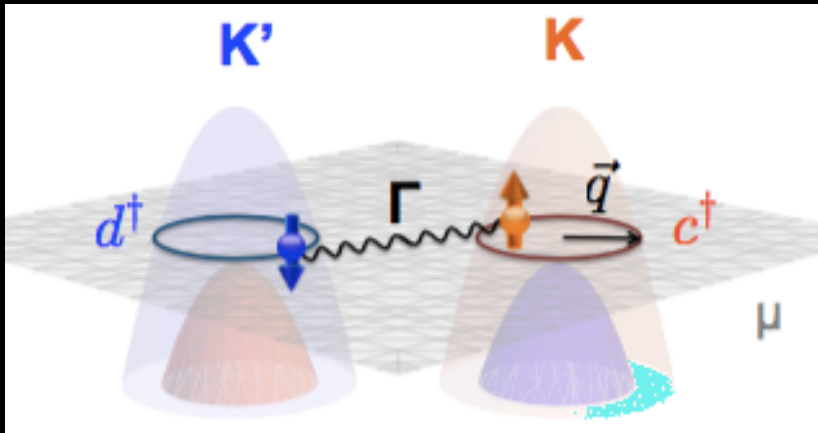
$$y \equiv \nu_0 \text{Log}(\Lambda_0/E)$$

- Divergence if  $\lambda^{(0)} < 0$

Two degenerate possibilities

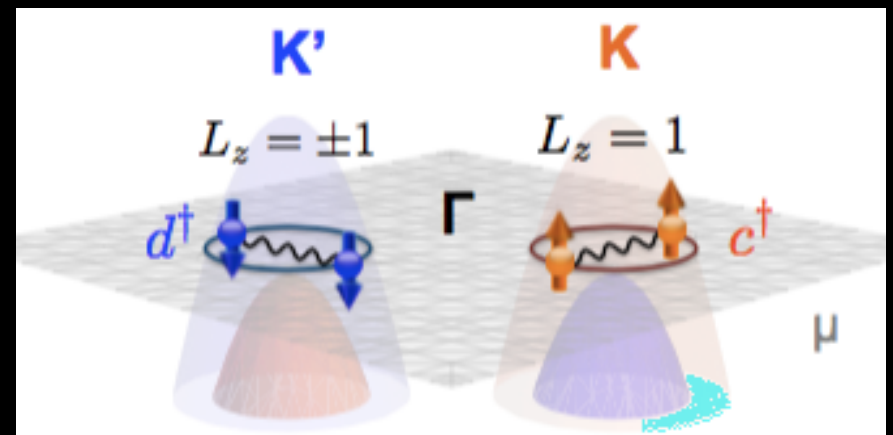
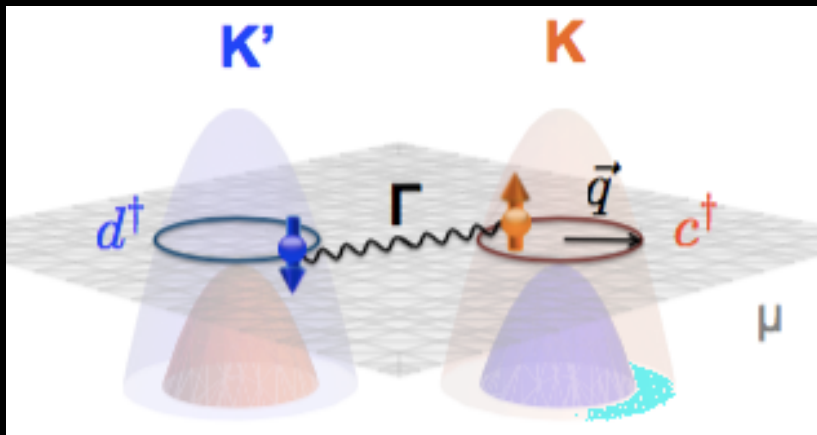
# Two degenerate possibilities

- Intra-pocket p+ip



# Two degenerate possibilities

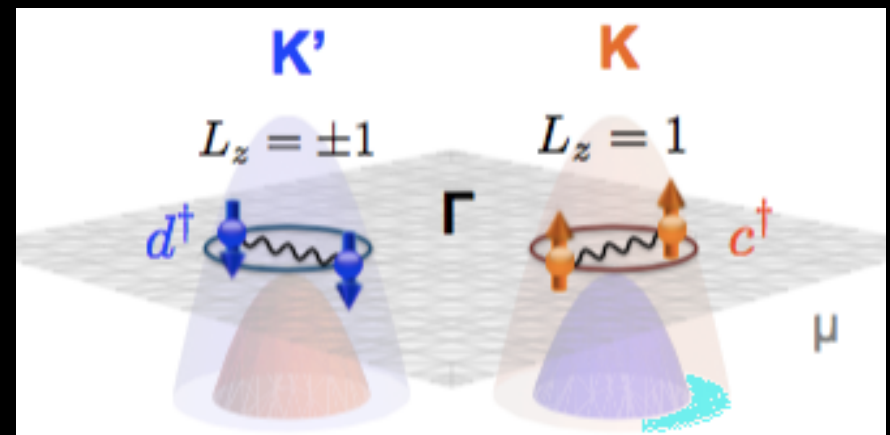
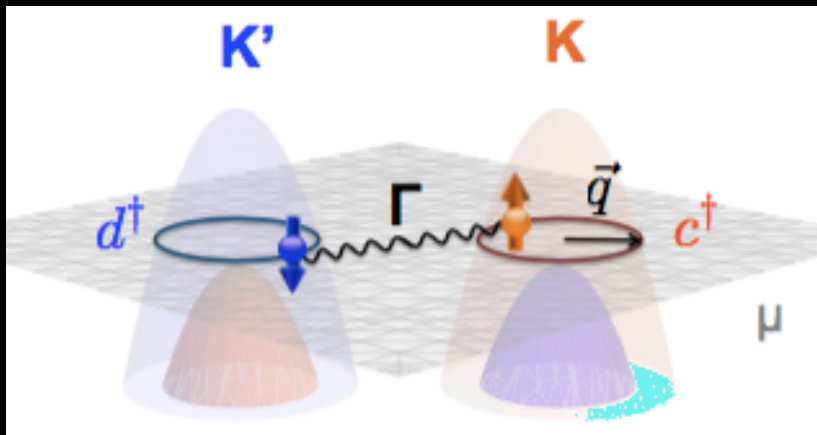
- Intra-pocket p+ip
- Inter-pocket p'wave





# Two degenerate possibilities

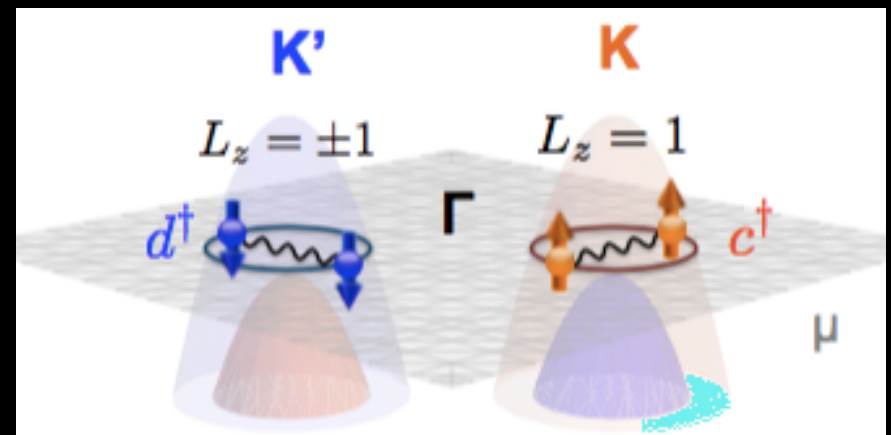
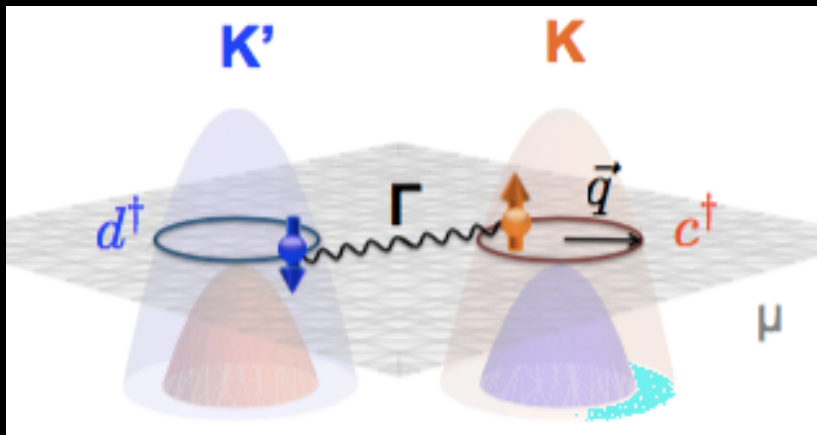
- Intra-pocket p+ip
- Inter-pocket p'wave



-T-breaking

# Two degenerate possibilities

- Intra-pocket p+ip
- Inter-pocket p'wave

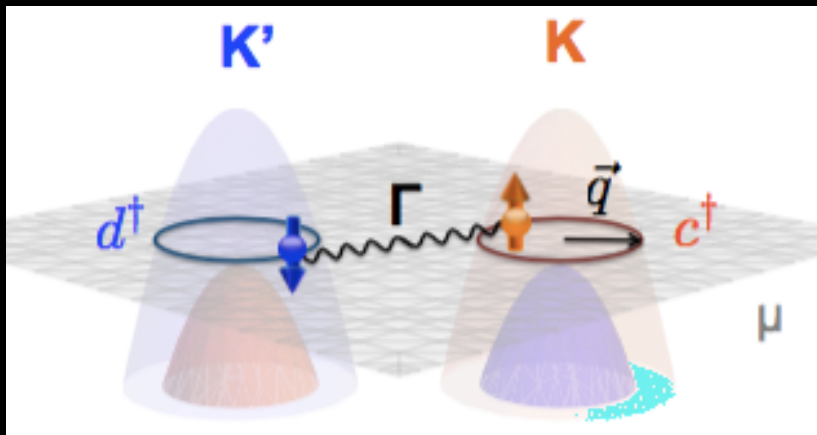


-T-breaking

-Analogous to  
 $\text{Sr}_2\text{RuO}_4$

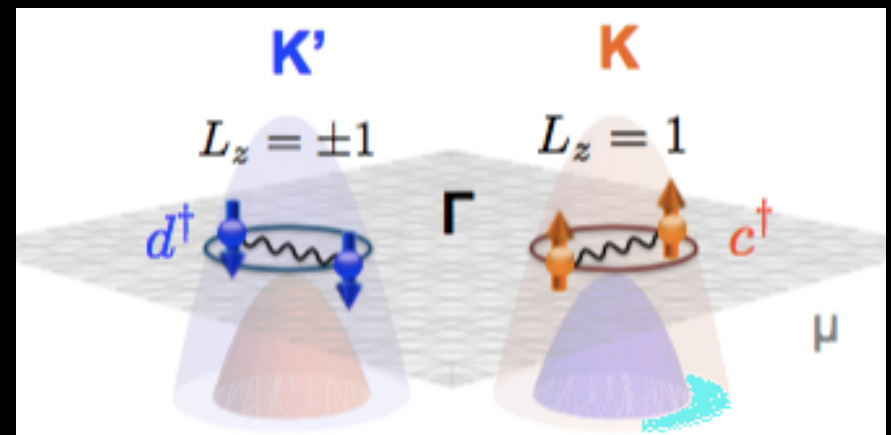
# Two degenerate possibilities

- Intra-pocket p+ip
- Inter-pocket p'wave



-T-breaking

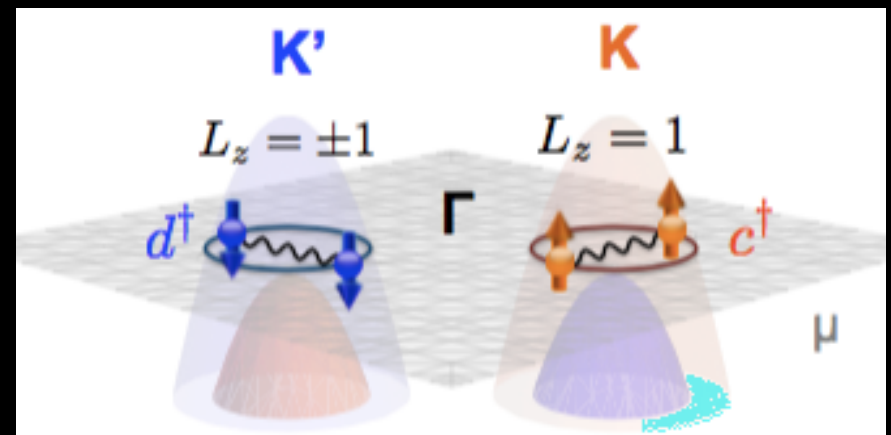
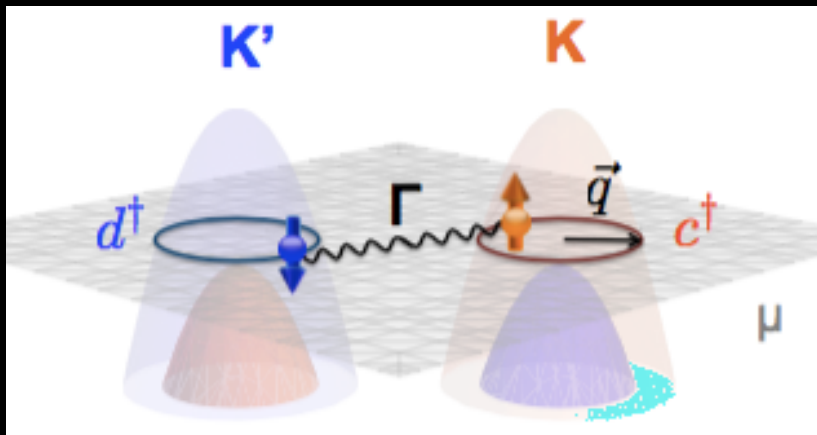
-Analogous to  
 $\text{Sr}_2\text{RuO}_4$



-Modulated

# Two degenerate possibilities

- Intra-pocket p+ip
- Inter-pocket p'wave



-T-breaking

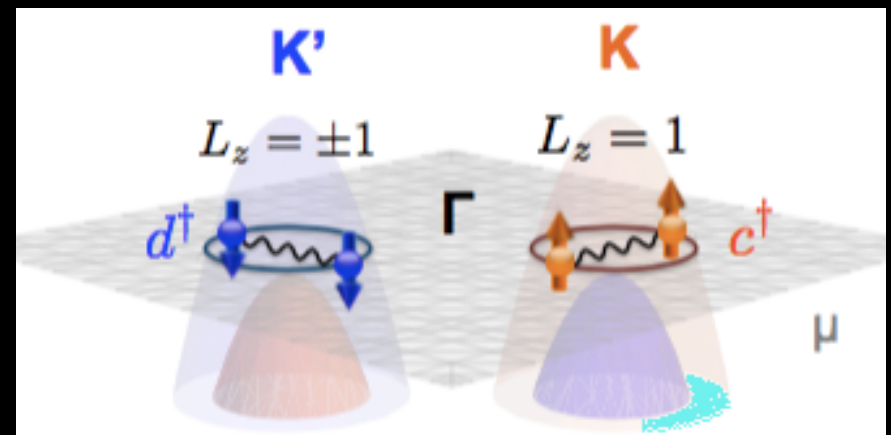
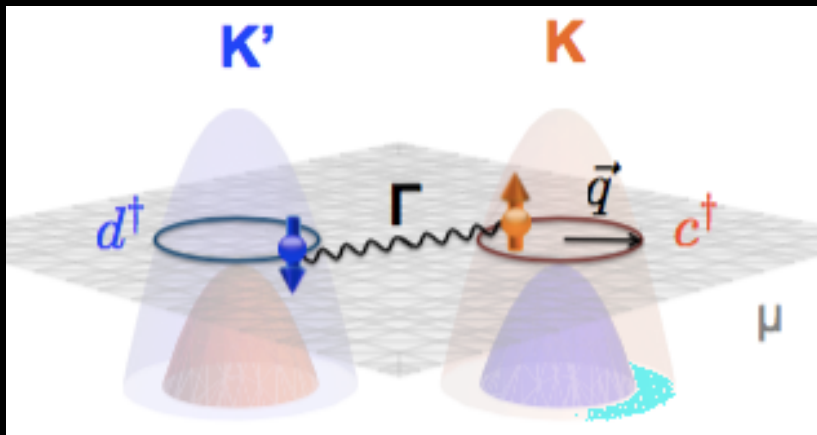
- C=1

-Analogous to  
Sr<sub>2</sub>RuO<sub>4</sub>

-Modulated

# Two degenerate possibilities

- Intra-pocket p+ip
- Inter-pocket p'wave

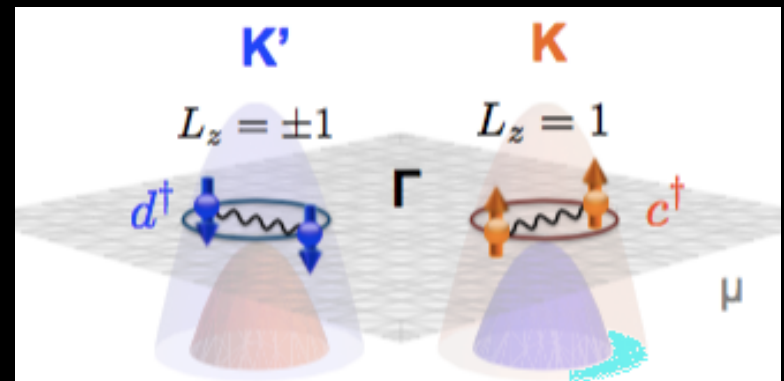
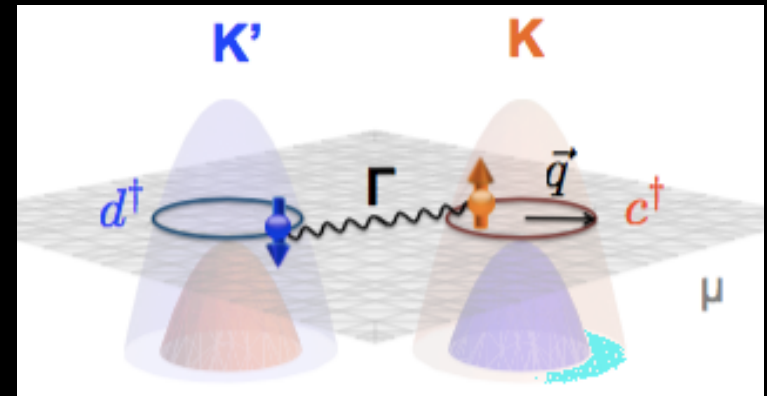
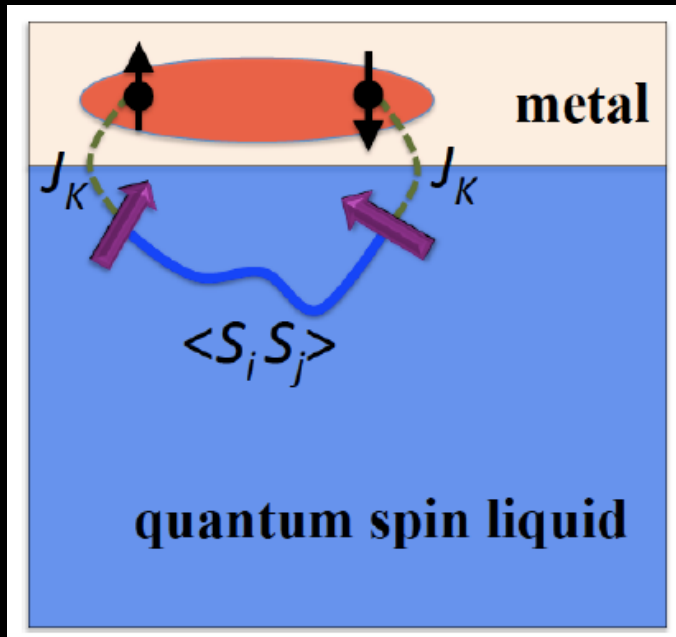


- T-breaking
- $C=1$
- Analogous to  $\text{Sr}_2\text{RuO}_4$

- Modulated
- $C \approx 1$  per pocket

# Designing 2D topological SC's

- Control interaction
- k-space spin split TMD



# Designing 2D topological SC's

- Control interaction
- k-space spin split TMD

